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# INVESTMENT, EQUIPMENT PRICES, AND INCOME TAX

By

John E. Ullmann
Stevens Institute of Technology

High income-tax rates and an increasing price level, prominent among the economic developments of the last 20 years, have long been strongly criticized in business circles. They are considered major deterrents to investment, acting as brakes on the growth of the economy. Demands for lower tax rates, and for a halt to price escalation (usually called inflation), are constant topics in the business press, in public addresses by high company officials,[1] in presentations for the justification of price or rate increases before regulatory commissions, and in institutional advertising designed for the general public.

Yet the last 20 years have seen an unprecedentedly high rate of investment, apparently uninhibited by the alleged handicaps. Clearly, there is a contradiction here, which, in view of the implications for economic policy and legislation, points up the need for studying the real and total effect of these factors upon the economy, i.e., for ascertaining the final consequences following upon the existing interrelationship of price changes and income tax.

The present study is based upon this premise: that the corporate income tax has served to reduce the opportunity or incremental cost of using equipment, because depreciation allowances are tax-deductible. It will be shown that, under the actual price movements and tax rates of the last 40 years, the total incremental cost of using equipment has been less than

[1]For a topical restatement of these views, see C. F. Hood, "From the Abacus to the Computer", Address to the 27th Annual National Meeting of The Controllers' Institute of America, Atlantic City, N. J., October 20, 1958. Mr. Hood was then the president of the United States Steel Corporation.

would have been the case if taxes and price changes were not taken into account. We do not, of course, argue that a company would not have been better off without having to pay increasing amounts for its equipment. The operative term here is incremental cost.

If the excess of revenues, over all costs other than depreciation, is taken as P, the tax on that would be tP where t is the tax rate. If an amount, D, can be allocated out of P for a tax-deductible depreciation expense, the tax payable is reduced to t(P-D). There is, in short, a tax saving of tD. The company has a choice. Either it pays D for its equipment, or it pays tD in taxes. The incremental cost of the first alternative is therefore (1-t)D. This reasoning is not restricted to 1 year, but may be extended, if necessary, to the whole depreciation period. The choice of the alternative of not buying the equipment may, of course, derive from a decision to make do with an existing item of machinery, or, simply, to refrain from the investment for any other reason.

Although this paper deals with the actualities of the past 40 years, the reasoning holds equally true for such extreme cases as no-tax or 100%-tax. If the tax rate were set at 100%, then, clearly, there is no incremental cost of installing equipment. The capital investment, in effect, comes straight out of government pockets. If, on the other hand, there were no tax, or depreciation allowances were not tax-deductible, the incremental cost of using the machinery would rise to D; the alternative would be for the company simply to keep the money and pay it to its shareholders, etc., just as it would with whatever is left after taxes now.[2] In any event, the present system of formalized de-

[2]In the recent hearings before the House Ways and Means Committee (reported in Business Week, December 5, 1959, p. 129), Professor R. Eisner suggested eliminating depreciation as an allowable deduction, and using the extra revenues to reduce the basic tax rates. Unlike present procedures, such a method would equalize cost and incremental cost, and would therefore probably act as a damper on investment.

preciation deductions is itself a creature of the tax laws, and did not become general in the United States until corporate-income taxation started. For purposes of this argument, the source of capital is also irrelevant; it may be from internal or external sources, just so long as the cost of the equipment is passed on to the users as periodic charges in the form of the D items.

Price changes would take their effect in the D items themselves, which, of course, are functions of time, assuming in other respects a replacement of machinery in kind. In the last 20 years, there has been a general upward trend in prices; if the last 40 years were considered instead, there would also be noted sustained periods of decreasing prices in the early 1920s and in the depression of the 1930s. There is recurring interest in the possibilities of computing depreciation allowances on a basis of replacement cost. This is not permitted by the income-tax laws at present. It has even been suggested that replacement value should be used in reporting net income to stockholders, even though not allowed for income-tax purposes.[3,4]

There are thus two parallel phenomena which will be investigated: First, the increase in tax rates has served to decrease the incremental cost of investment in depreciable items. Secondly, this decrease has been affected and, in recent years, partially offset, by corresponding changes (increases for the last 20 years) in the prices of machinery and equipment. Nevertheless, the net effect of these factors is a reduction in total incremental cost, relative to the cost computed without taking price

[3]American Institute of Certified Public Accountants: Opinion Survey on Price-Level Adjustment of Depreciation. New York, 1958. See also: Business Week, April 5, 1958, p. 79; and Business in Nebraska, University of Nebraska News, Vol. 38, No. 1, 1958.

[4]See Chemical Engineering, Vol. 65, No. 15, p. 70, 1958. Legal useful life is the amortization period for the equipment in question, as provided in the Internal Revenue Code, Bulletin F.

changes and taxes into account.

In examining the joint effect of these two developments, it is not intended to imply that one causes the other. Rather, they are independent factors, which have had their impact on what might otherwise be considered an "ideal" situation of no corporate taxes and a steady price level. It is true, of course, that, as aforesaid, businessmen tend to link the two in their complaints against economic trends affecting their enterprises.

Incremental cost, as here defined, is also frequently used in the justification of engineering alternatives. Its use and application have been described by Jelen[5], among others.

## PLAN OF STUDY

The first requirement is to compute the total incremental cost of using given equipment for the test period, at the costs and tax rates prevailing. This may be done in three stages. First, the total cost of equipment over the test period is computed, without taking price changes or taxes into account. Secondly, the price changes alone are considered. Thirdly, the tax rate is taken into account also, to arrive at the cumulative incremental cost.

If the test period is taken to start at year Y, the cumulative incremental cost thus refers to the sum of the annual costs over the test period, taking equipment-price changes into account, at current dollars, and using the tax rates in force. One suitable measure of such a series of investments is to be found in the present value at year Y of such a series, using a certain interest rate.

In this paper, periods of use of 40, 20, and 10 years, prior to December 31, 1957, are used. This approach is a modified

[5] F. C. Jelen: Chemical Engineering, Vol. 65, No. 15, pp. 116-8, 1958; Vol. 65, No. 2, pp. 123-8, 1958; Vol. 64, No. 9, pp. 271-5, 1957; Vol. 63, No. 6, pp. 247-52, 1956; Vol. 63, No. 5, pp. 165-9, 1956; and Vol. 61, No. 2, pp. 199-203, 1954.

version of the method of using capitalized costs for comparing the relative worth of alternatives, in that it considers a finite rather than an infinite number of replacements.

This requires attention to a second problem. As Grant has pointed out,[6] capitalized cost sometimes gives different results, according to what interest rates are used in the calculation. A second question is therefore: Do interest rates, or the "opportunity cost of money", have any effect on the relationship being investigated, when they are used in present-worth calculations?

A third problem which will be raised here is: Has there been any effect, as a result of equipment-price changes and income-tax rates, which favors or detracts from the use of longer-lived types of equipment? If tax rates and "decay rates" of the dollar are assumed constant for the indefinite future, then, according to Jelen[7], this tends to favor longer-lived equipment, with important implications both for management and manufacturers of equipment. It will be shown here that there appears to be little basis for generalizing on this topic. Hence, attention is drawn to the inadvisability of using tax rates and inflation as continuous phenomena. Compound interest accumulates inexorably, but tax rates and price changes are subject to political control, and hence variation.

As noted, the effects of price changes and income-tax rates will be considered for three periods of equipment-use: the 40 years from January 1, 1918 to December 31, 1957; the 20 years from January 1, 1938 to December 31, 1957; and the 10 years from January 1, 1948 to December 31, 1957. During each of these periods, it is assumed, three classes of equipment have been utilized: one lasting 10 years, one 5, and one which is replaced annually. It will be assumed that replacements are made in kind,

[6] E. L. Grant: Principles of Engineering Economy. The Ronald Press, New York, 1950. Third Edition. P. 118.

[7] Op. cit.

throughout the 40-, 20-, and 10-year periods, and that the life of the equipment actually equals the depreciation period. Straight-line depreciation will be assumed in all cases, since this has been the most common form used in the past.

At various times, during World War II and thereafter, accelerated write-offs were common. Since 1954, declining balance and sum-of-the-years-digits have been available as alternative methods. In each of them, to a varying degree, the asset is written off more rapidly during the early part of its life when its usefulness is greatest. There is also the hope that the writing-off can be done at a time of high tax rates, with subsequent net-income benefits when tax rates drop after the emergency. These hopes have, of course, been generally unjustified.[8]

As part of the flexibility implied by these accounting procedures, it is also important to consider the relationship between depreciation periods allowed and actual age of equipment. It has been shown that in a number of industries (metalworking, for example) equipment is often used well beyond the amortization period legally specified. However, possibilities of obsolescence of equipment have led to demands for wholesale reductions in legal useful lives.[9,10] In this study, chemical-equipment prices have been used as indicators of price change. In the chemical industry, known or predictable corrosion rates, scheduled and preventive maintenance, often give to the concept of equipment-life a concrete character which it lacks in other industries.

[8] Business Week, February 22, 1958, p. 57.

[9] A. A. Dettbarn: Depreciation and Obsolescence in the Automobile Industry. Master's Report in Industrial Engineering, Columbia University, 1950. See also surveys on the age of metalworking equipment by The American Machinist.

[10] Chemical Engineering, Vol. 65, No. 15, p. 70, 1958.

INVESTMENT ALTERNATIVES FOR ANALYSIS

In order to be able to judge the effects of equipment-price changes and tax rates by themselves, it will further be assumed that the original costs of the three alternatives are such that, without considering taxes and price changes, they will all be equal. That is to say, the present worth of the first cost and nine annual replacements of the equipment, which lasts 1 year, is the same as the first cost and one replacement of the 5-year equipment, and the same as the original cost of the 10-year equipment.

If $C_1$, $C_5$ and $C_{10}$ are the first costs of the 1-, 5-, and 10-year equipment, respectively, then

$$C_5 = C_1 \sum_{0}^{5} \frac{1}{(1+i)^n} \qquad (1)$$

$$C_{10} = C_1 \sum_{0}^{10} \frac{1}{(1+i)^n} \qquad (2)$$

The computations will be made in detail for an interest rate i of 8%, and the cumulative incremental cost for the whole period will also be derived for i=0%. If i=8% and $C_1$=\$100 (say), then $C_5$=\$431.20, and $C_{10}$=\$724.60. If i=0, then, of course, $C_5$=\$500 and $C_{10}$=\$1,000. In practice, of course, all the C's will change with time.

The prices of equipment have been derived here from the index of industrial-equipment costs prepared by Marshall and Stevens, Inc., of Chicago. Data for the chemical industry were utilized, although the index for it does not differ greatly from the Marshall and Stevens index for industry as a whole.[11]

[11] Chemical Engineering, Vol. 65, No. 4, pp. 143-4, 1958.

COMPUTATION

Let $m$ = life of equipment

$a$ = total number of pieces of equipment used; original + (a-1) replacements

$am$ = period of use

$C_m$ = first cost of equipment lasting m years, incurred at beginning of period of use

$I_j$ = price index of equipment ($I_0 = 1$ at time price of $C_m$ is first paid)

$t_j$ = federal corporation income-tax rate in $j^{th}$ year

$i$ = interest rate (opportunity cost of money)

$P = \frac{1}{1+i}$ (often called the discount factor)

$C_p$ = total cost over am years, taking price changes into account

$C_t$ = total incremental cost over am years

$C_c$ = total cost over am years, without taking price changes into account

Case 1: To find $C_p$

A machine costing $C_m$ at the beginning will cost $C_m I_{m+1}$ after the first m years are over, and a replacement is bought at the beginning of the $(m+1)^{th}$ year. The present value of this investment is

$$C_m I_{m+1} P^m$$

The second replacement will have a present value of

$$C_m I_{2m+1} P^{2m}$$

The last replacement, i.e., the $(a-1)^{th}$, will have a present value of

$$C_m I_{(a-1)m+1} P^{(a-1)m}$$

$C_p$ is the total of all these terms or

$$C_p = C_m \sum_{n=0}^{a-1} I_{mn+1} P^{mn} \qquad (3)$$

It will be noted that when I is set constant at 1, expression (3) is equivalent to (1) and (2).

Case 2: To find $C_t$

At the end of the first year, the tax credit due to the depreciation allowance will have served to reduce the cost to

$$C_m(1-\frac{1}{m}Pt),$$

since the tax credit will be based on an allowance of $\frac{C_m}{m}$ a year, under straight-line depreciation. The present value of this is $\frac{1}{m}Pt$. At the end of m years this becomes

$$C_m(1-\frac{1}{m}\sum_{x=1}^{m} t_x P^x)$$

At the end of 2m years this becomes

$$C_m \sum_{n=0}^{1} I_{nm+1} P^{mn}(1-\frac{1}{m}\sum_{x=mn+1}^{mn+m} t_x P^x)$$

The limits for x in this expression must deal with two separate cycles, one from year 1 to year m and the other from year m to year 2m. Summing this over am years, we have

$$C_t = C_m \sum_{n=0}^{a-1} I_{mn+1} P^{mn} (1-\frac{1}{m}\sum_{x=mn+1}^{mn+m} t_x P^x) \qquad (4)$$

Expression (4) may be reduced, as shown in Table 1, to give cumulative costs for the periods and interest rates indicated. The terms including $t_x$ are, of course, omitted in the calculations of $C_p$, which were also carried out.

The norm $C_c$, to which $C_p$ and $C_t$ must be compared, is, of course, given by the generalized version of expressions (1) and (2), or by setting I=1 in (3).

$$C_c = C_m \sum_{n=0}^{a-1} P^{mn} \qquad (5)$$

Table 2 gives the price indexes derived for the three periods of use, as well as the tax rates in effect at the time. The standard rate was taken in all cases, except that, in years where excess-profit tax was in effect, the actual rate was

TABLE 1

FORMULAE USED IN COMPUTATIONS OF TOTAL INCREMENTAL COSTS $C_t$

$(i=8\%\ P=\frac{1}{1.08}=0.9259)^*$

| am $^m$ | 1 | 5 | 10 |
|---|---|---|---|
| 40 | $C_1 \sum_{0}^{39} I_{n+1} P^n (1-t_n P)$ | $C_5 \sum_{0}^{7} I_{5n+1} P^{5n} (1-\frac{1}{5} \sum_{5n+1}^{5n+5} t_x P^x)$ | $C_{10} \sum_{0}^{3} I_{10n+1} P^{10n} (1-\frac{1}{10} \sum_{10n+1}^{10n+10} t_x P^x)$ |
| 20 | $C_1 \sum_{0}^{19} I_{n+1} P^n (1-t_n P)$ | $C_5 \sum_{0}^{3} I_{5n+1} P^{5n} (1-\frac{1}{5} \sum_{5n+1}^{5n+5} t_x P^x)$ | $C_{10} \sum_{0}^{1} I_{10n+1} P^{10n} (1-\frac{1}{10} \sum_{10n+1}^{10n+10} t_x P^x)$ |
| 10 | $C_1 \sum_{0}^{9} I_{n+1} P^n (1-t_n P)$ | $C_5 \sum_{0}^{1} I_{5n+1} P^{5n} (1-\frac{1}{5} \sum_{5n+1}^{5n+5} t_x P^x)$ | $C_{10} (1-\frac{1}{10} \sum_{1}^{10} t_x P^x)$ |

*For i=0 P=1

TABLE 2

PRICES OF EQUIPMENT AND CORPORATE INCOME TAX RATES[12,13]

| Year | Price Index of Chemical Industry Equipment (a) (1918=100) | Standard Corporate Tax Rate (b) Per Cent | Year | Price Index of Chemical Industry Equipment (a) (1918=100) | Standard Corporate Tax Rate (b) Per Cent |
|---|---|---|---|---|---|
| 1918 | 100 | 12 | 1938 | 75 | 21 |
| 1919 | 109 | 10 | 1939 | 75 | 20 |
| 1920 | 135 | 10 | 1940 | 75 | 32 |
| 1921 | 104 | 10 | 1941 | 83 | 49 |
| 1922 | 75 | 12.5 | 1942 | 91 | 57 |
| 1923 | 84 | 12.5 | 1943 | 92 | 59 |
| 1924 | 94 | 12.5 | 1944 | 94 | 60 |
| 1925 | 94 | 13 | 1945 | 95 | 60 |
| 1926 | 89 | 13.5 | 1946 | 113 | 40 |
| 1927 | 87 | 13.5 | 1947 | 135 | 40 |
| 1928 | 87 | 12 | 1948 | 147 | 40 |
| 1929 | 83 | 11 | 1949 | 147 | 41 |
| 1930 | 73 | 15 | 1950 | 152 | 50 |
| 1931 | 70 | 16 | 1951 | 161 | 62 |
| 1932 | 69 | 24 | 1952 | 162 | 61 |
| 1933 | 63 | 17 | 1953 | 164 | 61 |
| 1934 | 67 | 17 | 1954 | 166 | 54 |
| 1935 | 70 | 18 | 1955 | 171 | 52 |
| 1936 | 74 | 19 | 1956 | 187 | 52 |
| 1937 | 79 | 20 | 1957 | 202 | 52 |

(a) Converted from 1926 = 100 in Footnote 13.
(b) Except when excess profit tax was in effect. Actual ratio taxes/gross earnings was then used.

[12]Bureau of Internal Revenue: Statistics for Income for 1947, Part II. United States Government Printing Office, 1953, 470 (and later issues).

[13]Chemical Engineering, Vol. 65, No. 4, pp. 143-4, 1958.

computed by taking the ratio of taxes paid to gross income.

## ANALYTICAL FINDINGS

When the total incremental costs $C_t$ are computed as outlined, it is found that they are in all cases less than $C_p$ and $C_c$, the total costs before taxes, with and without price changes. This is illustrated in Table 3, where each $C_t$ may be compared to its corresponding $C_p$ or $C_c$ value, independently of total use period or of interest rate. To bring these relationships into sharper focus, Table 4 has been prepared, which gives an index of $C_t$ and $C_p$ in terms of $C_c$ = 100. It then becomes clear that, over a 40-year period, $C_p$ and $C_c$ are actually quite similar, periods of rising and falling prices tending to cancel each other out. The actual values appear to be determined by the interest rate here. If approximately 4% had been used instead, the results would probably be even more similar.

What is really important here, however, is the fact that, though the values of $C_p$ have increased greatly compared with $C_c$, in the 20- and 10-year periods of price rise, these changes have been more than eliminated when $C_t$, the cumulative incremental costs, were considered instead. Moreover, the $C_t$'s represent sharp reductions below the $C_c$ norm, except in the 20-year period, during which, as noted, prices rose consistently. This is, perhaps, the "worst" period that could have been chosen, from that viewpoint.

Again, these conclusions are independent of the interest rate. Actually, the factor P reduces the effect of price changes later in the use period, particularly where this is long. P is less than 1, and higher powers tend towards zero. Therefore, if an interest rate higher than 8% is considered more appropriate, these effects would only be enhanced.

The periodic increases in $C_c$, $C_p$ and $C_t$ may be represented graphically in the form of a stepped curve reaching a plateau at the end of the use period given by the final figures of Table 4. This, as shown in Figures I, II and III (see pp.19 , 20 , and 21), may be used to demonstrate the differences between $C_c$, $C_p$

TABLE 3

SUMMARY OF $C_c$, $C_p$ AND $C_t$ COSTS OVER VARIOUS PERIODS OF USE
(With and Without Interest)

| Int. Rate (%) | Life of Equipt. (Yrs.) | First Cost $ | Period of Use - Cumulative Values, Dollars | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1918-1957(a) | | | 1938-1957(a) | | | 1948-1957(a) | | |
| | | | $C_c$ | $C_p$ | $C_t$ | $C_c$ | $C_p$ | $C_t$ | $C_c$ | $C_p$ | $C_t$ |
| 8 | 1 | 100 | 1288 | 1222 | 1006 | 1060 | 1594 | 899 | 725 | 801 | 417 |
| 8 | 5 | 431.20 | 1288 | 1175 | 996 | 1060 | 1493 | 968 | 725 | 760 | 484 |
| 8 | 10 | 724.60 | 1288 | 1239 | 1077 | 1060 | 1383 | 951 | 725 | 725 | 473 |
| 0 | 1 | 100 | 4000 | 4295 | 2767 | 2000 | 3454 | 1733 | 1000 | 1128 | 533 |
| 0 | 5 | 500 | 4000 | 4060 | 2632 | 2000 | 3210 | 1609 | 1000 | 1060 | 503 |
| 0 | 10 | 1000 | 4000 | 4090 | 2728 | 2000 | 2960 | 1501 | 1000 | 1000 | 480 |

(a) Both years inclusive

TABLE 4

RATIO INDEXES OF $C_t$ AND $C_p$ TO $C_c$
FOR VARIOUS USAGE PERIODS AND INTEREST RATES

| Int. Rate (%) | Life of Equipt. (Yrs.) | Period of Use - Ratios | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 40 Yrs. 1918-1957 (a) | | | 20 Yrs. 1938-1957 (a) | | | 10 Yrs. 1948-1947 (a) | | |
| | | (b) | (c) | (d) | (b) | (c) | (d) | (b) | (c) | (d) |
| 8 | 1 | 100 | 95 | 78 | 100 | 150 | 85 | 100 | 110 | 58 |
| 8 | 5 | 100 | 91 | 77 | 100 | 141 | 91 | 100 | 105 | 67 |
| 8 | 10 | 100 | 96 | 83 | 100 | 130 | 90 | 100 | 100 | 65 |
| 0 | 1 | 100 | 107 | 69 | 100 | 173 | 87 | 100 | 113 | 53 |
| 0 | 5 | 100 | 102 | 66 | 100 | 160 | 80 | 100 | 106 | 50 |
| 0 | 10 | 100 | 102 | 68 | 100 | 148 | 75 | 100 | 100 | 48 |

(a) Both years inclusive

(b) $C_c$

(c) $C_p/C_c$

(d) $C_t/C_c$

and $C_t$, as they build up with time. As expected, the heavy solid line, $C_t$, is always lower than the others. Only the details for the 1-year replacement cycles are shown, to simplify the presentation. A similar result would be obtained for the others, except for the larger steps in the curves.

In Figure I, the effect of alternating periods of price-rise-and-decline is reflected in the relative positions of the $C_c$ and $C_p$ curves. The lower prices of the 1930s evidently caused the widening difference between the two, which was then gradually closed as prices increased during the 1940s and 1950s. It may be expected that the small excess of $C_c$ over $C_p$ will be eliminated in about 5 to 7 years if present trends continue. However, as stated before, this whole difference depends upon the interest rate selected. When the appropriate results were computed for the condition of no-interest, the crossover period occurred in 1953, after which $C_p$ exceeded $C_c$, as indicated in Tables 3 and 4.

No clear-cut advantage of longer-lived equipment emerges from this analysis. If the 1-year alternatives are taken as 100 for $C_c$, $C_p$ and $C_t$, then, as shown in Table 5, a rather varied set of results is obtained, depending upon total use cycle, as well as interest rates. When these considerations are taken together with the operational uncertainties in equipment life, it would be doubly risky to attempt any major conclusions from the data.

## CONCLUSIONS

We have shown that there has been a net decline in the cumulative incremental cost of using equipment, relative to the cost measured without taking taxes or price changes into account. We have also demonstrated that this finding is independent of interest rates used in present-worth computations, life span of equipment, and number of renewal cycles within the last 40 years. Since incremental costs play such an important part in the justification of investment, these conclusions offer an explanation for the demonstrable coexistence of high

TABLE 5

RATIOS OF CUMULATIVE COSTS FOR 5- AND 10-YEAR EQUIPMENT TO THOSE OF 1-YEAR EQUIPMENT

| Int. Rate (%) | Life of Equipt. (Yrs.) | Period of Use - Ratios | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 40 Yrs. 1918-1957 (a) | | | 20 Yrs. 1938-1957 (a) | | | 10 Yrs. 1948-1957 (a) | | |
| | | (b) | (c) | (d) | (b) | (c) | (d) | (b) | (c) | (d) |
| 8 | 1 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 8 | 5 | 100 | 88 | 99 | 100 | 94 | 108 | 100 | 95 | 116 |
| 8 | 10 | 100 | 101 | 106 | 100 | 87 | 106 | 100 | 90 | 114 |
| 0 | 1 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 0 | 5 | 100 | 95 | 95 | 100 | 93 | 93 | 100 | 94 | 94 |
| 0 | 10 | 100 | 95 | 99 | 100 | 86 | 87 | 100 | 89 | 90 |

(a) Both years inclusive

(b) $C_c = 100$

(c) $C_{pj}$ ; $C_{p1} = 100$

(d) $C_{tj}$ ; $C_{t1} = 100$

rates of investment, increasing equipment prices, and high income-tax rates. The findings confirm the observations of Melman,[14] who noted that high prices and tax rates did not hinder the widespread mechanization of production facilities. Melman's examples are, in part, British; British tax rates and price increases were even higher than those in the United States. Albin made similar observations in a study of replacement policies in

[14] S. Melman: Dynamic Factors in Industrial Productivity. John Wiley and Sons, New York, 1956. Pp. 56-7.

a United States metalworking industry.[15]

Certainly business is worse off in a time when equipment prices are rising than it would be if they were steady or declined.[16] In relation to the incremental cost and the present tax rates, however, steady prices would mean chiefly that the divergence shown between $C_c$ and $C_p$ on the one hand (equal with constant prices, of course) and $C_t$ on the other, would be even wider (cf. Figure I). In effect, the reduction in incremental cost of equipment tends to encourage capital investment, in that it reduces the amount which must be justified by the anticipated return, and thus increases the rate of return on incremental investment.

Under these circumstances, moreover, increased costs of equipment need not be entirely passed on to the consumer in the form of higher prices. High prices of machinery and high taxes are not, therefore, critical factors in the purchase of machinery, compared with factors such as the replacement of manpower.[17] Additional outlays for machinery can then be justified, as Melman

[15] G. Albin: An Analysis of Criteria for Capital Equipment Replacement. Master's Report in Industrial Engineering, Columbia University, 1956.

[16] A recent study prepared for the Joint Economic Committee, United States Congress (T. A. Wilson: "The Analysis of Inflation in Machinery Prices", Study Paper No. 3, November 6, 1959), notes that "the erosion of the capital purchasing power of personal savings, and the lessening of competition resulting from a capital-goods inflation, may have inhibiting effects upon the growth potential of the economy". This paper deals mainly with the price rises from 1954 to 1958. Since the tax rates remained about the same during that time, any increase in cost of machinery would have been reflected in a corresponding change in incremental cost. Such a trend, if continued, would of course, tend to reduce the gap between $C_c$ and $C_t$.

[17] This, of course, does not justify high American machinery prices, which have, in part, eliminated the industry in international competition.

has shown,[18] mainly on the basis of the alternative costs of man-hours and machine-hours.

During the past 40 years, productivity of man-hours and of raw materials has shown a consistent upward trend.[19] This often means that higher output is realized from the same production equipment. The increase in capital productivity realized in many industries would widen the spread between $C_c$ and $C_t$ even more, if it were applied to the latter.

At any rate, the increasing tax rates of the past 40 years have served to reduce the total incremental cost of machinery, even in the presence of a sustained price increase over the last half of that period. A drastic liberalization of the depreciation laws would, in view of this, take on more of the role of a subsidy to capital-intensive industries. This may or may not be an entirely worthy method of stimulating the economy, but it should be recognized for what it is, rather than treated as the alleviation of a particular injustice of the tax laws.

The negative side of this argument is reinforced by a recent study in Business Week.[20] It was found that rising depreciation allowances in the next 4 years will "throw off a record amount of cash into company treasuries". Though these funds would normally be earmarked for more expansion, planned expenditures for new plant are lower than the expected inflow. Accordingly, the companies "could decide to reduce their debts, increase research and advertising, acquire other companies, improve dividends, salaries and wages, or worker benefits". From this, it would seem that the complaints, against the tax and price

[18] Op. cit.

[19] Joint Economic Committee: Productivity, Prices and Incomes. 85th Congress, First Session. United States Government Printing Office, Washington, D. C., 1957. Pp. 89-92.

[20] Business Week, May 16, 1959. P. 47.

phenomena described, result, to a major extent, from a desire on the part of management to maximize internal financing.

* * *

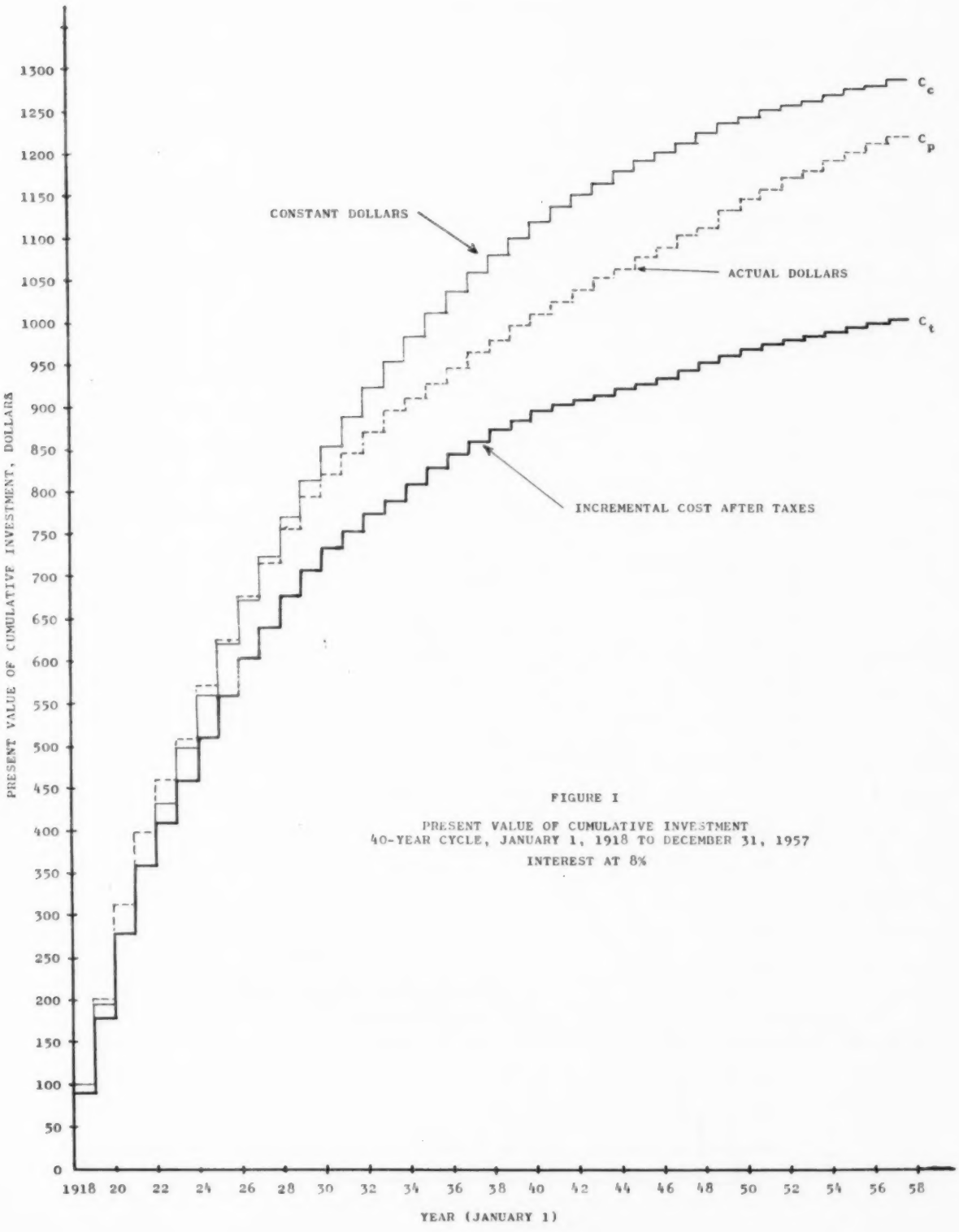


FIGURE I

PRESENT VALUE OF CUMULATIVE INVESTMENT
40-YEAR CYCLE, JANUARY 1, 1918 TO DECEMBER 31, 1957
INTEREST AT 8%

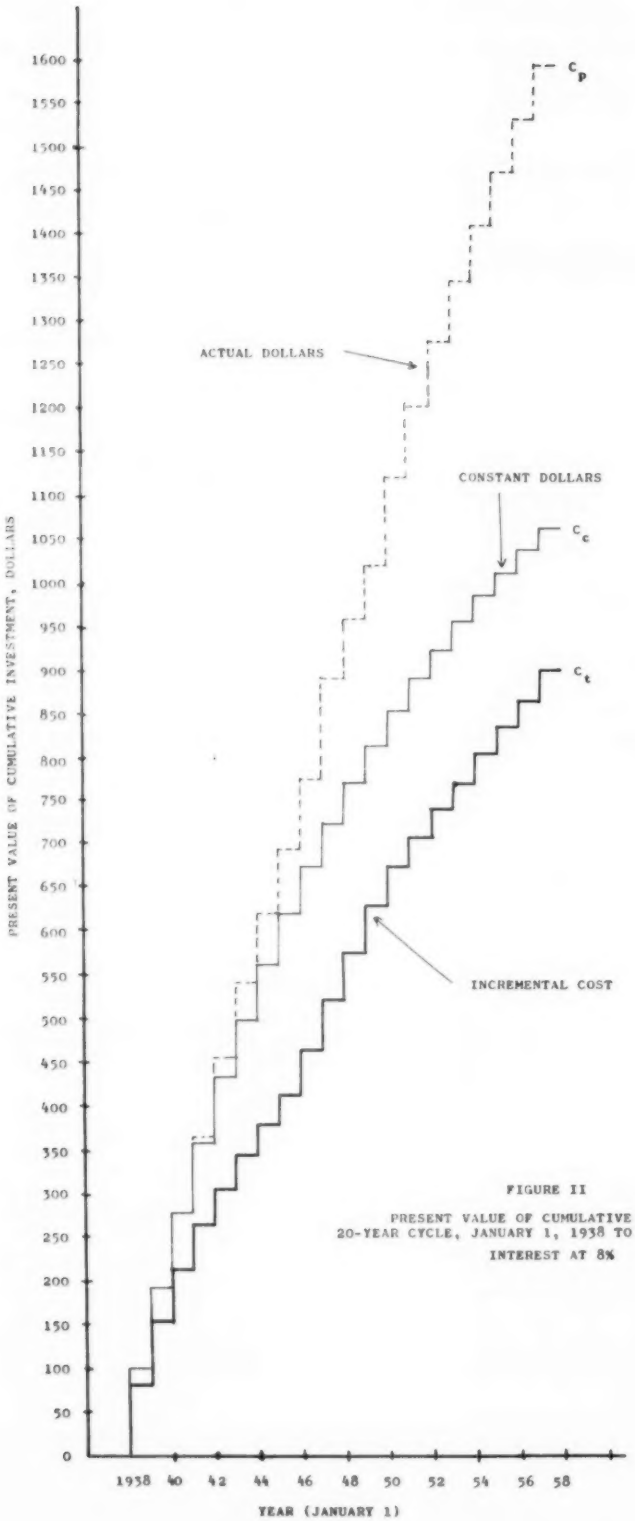


FIGURE II

PRESENT VALUE OF CUMULATIVE INVESTMENT
20-YEAR CYCLE, JANUARY 1, 1938 TO DECEMBER 31, 1957
INTEREST AT 8%

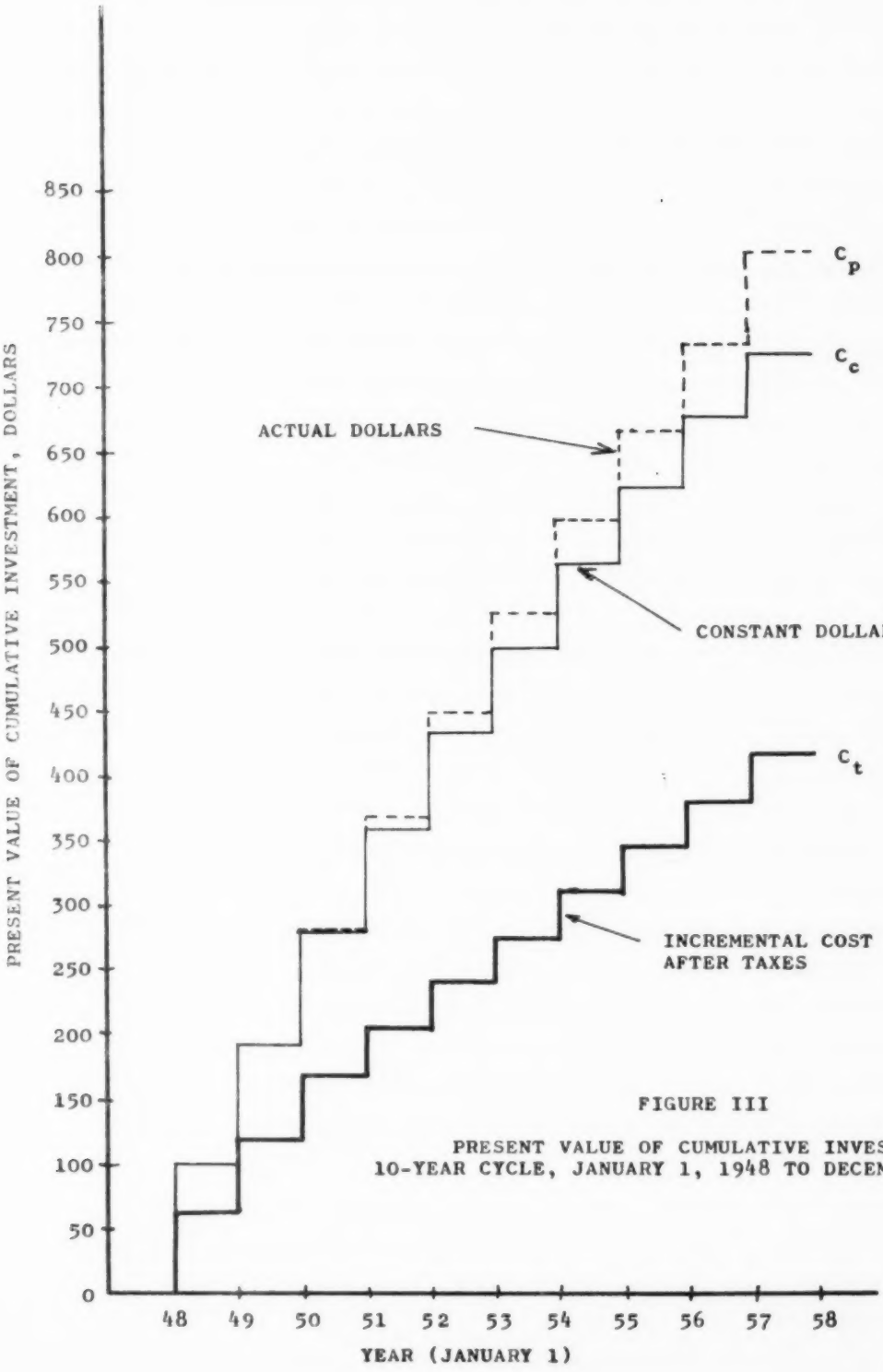


FIGURE III

PRESENT VALUE OF CUMULATIVE INVESTMENT
10-YEAR CYCLE, JANUARY 1, 1948 TO DECEMBER 31, 1957

# MANAGEMENT'S YARDSTICKS OF PROJECT JUSTIFICATION IN THE CHEMICAL INDUSTRY

By

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In my previous article on "Project Justification in a Multiproduct Chemical Plant", which appeared in the Fall 1959 issue of The Engineering Economist, I discussed the underlying causes for equipment acquisition and replacement in the chemical industry, and followed with a presentation of the payout-time technique for evaluating capital expenditure, as designed by a leading dye and pharmaceutical plant. Many other well-known techniques, such as the Annual Cost method, the Discounted Cash Flow, and the MAPI approach, are also available to management. However, none of the techniques mentioned relieves management of the basic responsibility for making the ultimate decisions regarding capital acquisition for expansion or replacement. The methods do present management with an analysis pointing up the relative merits of a given type of investment, and give some indication as to which alternative is the more desirable. Management, nevertheless, must assume responsibility for the final decision.

Many equipment and replacement decisions impose a long-term commitment on the company. Any error, in any of the decisions, can be costly to the company. Profit, then is the payment for sound decision making.

In making such decisions, actually, management is faced with two alternatives: old machines can either be retained to keep

*The material used for this article (and for "Project Justification in a Multiproduct Chemical Plant", which appeared in The Engineering Economist, Vol. 5, No. 2) was obtained while working for a doctoral degree at the Illinois Institute of Technology, under the guidance of Professor Gerald J. Matchett.

down the capital cost, at the expense of accepting higher operating costs; or they can be replaced by a new machine to obtain temporary mechanical perfection, at the expense of an increase in capital cost. A decision must be made.

As a practical matter, sound judgment depends upon standards of measurement. In an attempt to arrive at the right decision at the right time, a number of yardsticks are needed for assisting and guiding management -- in making final decisions -- which, taken together, form the basis for a sound conclusion. The following yardsticks are not listed in order of importance, but as they will be discussed.

(1) Availability of funds
(2) Market potential
(3) Patentability
(4) Payout time
(5) Allowable faster write-off period
(6) Getting into the business
(7) Getting out of the business

When a project involving a capital expenditure is proposed to management, a consideration of any one, or a combination of, these yardsticks, determines the final fate of the project.

## AVAILABILITY OF FUNDS

The real starting point, in determining whether or not to invest capital in new equipment, is the availability of funds. Unless the plant has the funds on hand, or is sure that funds can be acquired when needed, none of the capital projects will materialize. The availability of funds also controls the size and the scope of the capital investment that the plant can undertake. Often, some of the proposals, although shown to be highly profitable, are temporarily delayed because of the large sum of new capital involved.

For a plant with a budget-minded management, the amount of money available is usually based on the estimated yearly budget. This tends to establish the upper ceiling of the total allowable capital expenditures for the budget period. Thus, management must select, from among many replacement opportunities, those

offering the best profit potentials, so that the total expenditure can be kept within the ceiling specified by the budget.

MARKET POTENTIAL

If the funds are available, the next test in evaluating a proposed project is an estimate of the potential market. Factors which should be considered in estimating market potential are:

(1) Effect on established product lines within the company

(2) Market trend

(3) Selling price of the product made from the proposed project

In checking over these factors, one can see that they are relevant to a decision involving added investment in new equipment. Certainly, management will be interested in the effect of a proposal on the established product lines within the company. Even though the initial responsibility, for selecting and recommending equipment, generally rests with the engineering group of the company, management should insist that the engineering group consult with the operating departments, as to potential markets and profits, and as to whether a proposed product, for which the equipment is recommended, would fit logically into the company's product line. For example, assume that a plant is primarily a producer and marketer of bulk chemicals, which are sold to other chemical companies, and does not engage in marketing consumer products directly. Consequently, any proposal, which involves the installation of equipment to convert products to consumer items, is most likely to be rejected.

The second factor, in measuring market potential, is the market trend. Management must evaluate this information before making the decision. It is generally true that the ups and downs of the general market trend affect the expected sales volume, even of the highly specialized items that may be produced by the company. Such fluctuations, of course, must be considered, in arriving at an upper ceiling on the return that may be expected

on the investment. However, estimates should be based only on the expected general market trend of the product, and should not include short-run seasonal variations.

Finally, expected return cannot be calculated without estimates of unit price and unit volume. When the installation of equipment for a completely new product is under study, the estimates of price and volume often become exceedingly difficult. On the other hand, it could be catastrophic to find, after equipment had been installed, that the new product, so hopefully proposed by the research group and highly recommended by the engineering department, has only limited utility, or is rendered obsolete by new or superior competitive products. The rapidly changing technological and economic climate, which a chemical plant has to face today, should force management to consider carefully the expected price, volume, and estimated market life of the proposed product, before the final decision is made.

## PATENTABILITY

The patent situation is another important factor affecting the decision on the installation of equipment. The decision (as to whether or not a project should be undertaken) is often influenced by the absence, or existence, of broad or narrow patents. If no patents block the project, patentability of the process may be a factor affecting the economic justification of the project. Licensing policy, for a licensee or licensor, may likewise affect the decision to invest money in equipment. No new project should be approved unless consideration is given to the patent situation.

## PAYOUT-TIME REQUIREMENT

There may be upper or lower limits, within which a percentage or dollar figure reflecting payout must fall. However, there is no set rule to limit payout time for the various capital investments. Nevertheless, the payout time calculated does, in a major way, affect management's final decision in the chemi-

cal industry. The present trend appears to be that those projects which can be paid off within two or three years are usually favored. Generally, a comparatively long payout time might be acceptable for expansion in the production of an established product, whereas fast payout might be essential for a new product, in a new line where rapid change of product, and thus, obsolescence, are to be expected. The payout time also depends upon the size and the scope of theproject. A longer payout time might be acceptable for projects of a minor nature, involving only a small expenditure.

## ALLOWABLE FASTER WRITE-OFF PERIOD

At the popular 10% depreciation rate per year, in the chemical industry, a comparatively long payout time often results, and this in turn may result in rejection of the project. Sometimes this situation can be altered by using shorter write-off periods, with permission from the Office of Defense Mobilization. A shorter write-off period can be applied only to those projects involving the national defense, or to those having high rates of obsolescence. The shorter write-off period will provide a larger amount of capital recovery per year, and result in a shorter payout time on the capital investment.

## GETTING INTO OR OUT OF THE BUSINESS

In addition to the five major yardsticks above mentioned, there are also two others which should be taken into consideration by management, prior to making a final decision. These are getting into the business and getting out of the business.

Getting into the business, at the strategic moment, sometimes overrules some of the other yardsticks, as long as the company has the funds available for the project. This is especially true for those products or processes, newly developed by the research department, which appear extremely promising for the development of future markets.

Getting out of the business also influences the final de-

cision of management on equipment replacement. For instance, the sales of a product may be diminishing, due to product obsolescence. The plant expects to discontinue production of this product entirely, in the near future. Under this condition, any new capital addition, or replacement of the major existing equipment, on this unit, is almost certain to be rejected by management, even though the computed savings may be significant.

## GENERAL CONCLUSION

The seven yardsticks that management uses in evaluating equipment and replacement proposals do not apply equally in making each decision. For some investment decisions, several of the yardsticks may be irrelevant. For some, one of the yardsticks may have overwhelming influence, overriding undesirable factors. For certain others, all may have a bearing. In any event, the final decision should represent the collective judgment of management -- a judgment arrived at after applying the appropriate yardsticks to the particular investment proposal.

* * *

# ECONOMICS IN AMERICAN ENGINEERING SCHOOLS

By

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> "Then the political economists may meet together and discuss the improved social order, burn their old books, and erect a monument to the man who above all others contributed the means for obtaining the wealth of nations, James Watt, the engineer."
>
> - William Kent, 1895

## INTRODUCTION

History records that political economy received at least perfunctory attention in what were then engineering schools, over one hundred and thirty years ago. In 1827, Valentine B. Horton, who was later to become a Whig congressman and to make and lose a fortune, became "a professor of Moral Philosophy and Political Economy" at the American Literary, Scientific and Military Academy, the forerunner of Norwich University.[1] During the 1840s and '50s, many engineering schools included a few lectures in moral philosophy and political economy in their programs. More formal and more enthusiastic promotion of economics on engineering campuses was evident under the aegis of Francis Amasa Walker, a noted economist and statistician, who, after service at the Sheffield School at Yale, became president of Massachusetts Institute of Technology in the early 1880s. Under his guidance, and that of Davis R. Dewey, who became an assistant professor of Economics and Statistics at M.I.T. in 1888, economics began to flourish on at least one engineering campus.

[1] W. A. Ellis: Norwich University, 1819-1911. The Capital City Press, Montpelier, Vermont, 1911. Volume I, p. 10.

Formal attention to economics by the Society for the Promotion of Engineering Education dates from about 1916.[2] Since then, economics has gained substantial importance in engineering programs, despite a general decline in the socio-humanistic content of engineering education, which seems to have been arrested only since World War II.

But what of economics on contemporary engineering campuses? What is its status at mid-century? In order to obtain answers to these questions, a sizable questionnaire was circulated throughout the United States, as part of a comprehensive, evolutionary study of "economics for engineers".[3] The returns were processed in 1956-57, and, along with a search of the literature of engineering education, they form the basis of the observations which follow.

In the course of the investigation, all schools with E.C.P.D. accreditation (150 at that time) were contacted, with the gratifying result that some sort of response was received from 145 (or 96.7%) of the total. The questionnaire posed questions on some 23 aspects of economics in the engineering-school environment, but in view of the brevity of this report, only the more salient findings are here reported. Specifically, only those data are included which shed light on the nature and extent of an economics requirement, the aims thereof, the use of special courses and special methods, and the degree of integration with other social sciences.

One further word of explanation is in order. Most of the items on the questionnaire were phrased so as to elicit either a yes-no or multiple-choice answer. Therefore, in most instances, answers were tabulated on a per-cent-of-schools basis. However,

[2]Society for the Promotion of Engineering Education: "Preliminary Report of Committee No. 14", Bulletin 6 , June 1916, p. 723.

[3]E. J. Holstein: The Development of Thought Concerning Economics Instruction in American Engineering Colleges. Microfilm. University Microfilms, Inc., Ann Arbor, Michigan, 1957.

when it was logical for the respondent to check more than one answer (for example, when -- as is usually the case -- more than one aim was being pursued), results were expressed as percentage frequencies. Furthermore, although 96.6% of the schools responded, all schools did not answer all questions. Therefore, response percentages are noted in the tables for each individual question.

For purposes of analysis, the schools were divided into four categories, according to (1) age, (2) size, (3) public or private status, and (4) geographic location. A school was considered old if it was generally recognized as a school of engineering prior to 1893, and new if it was so recognized after 1893. It was considered small if it had recently granted under 50 first engineering degrees, medium if between 50 and 200, and large if over 200. The source of income and control determined if it was public or private, with schools sponsored by religious groups classified as private. Locationwise, schools were designated as North-East-Central (north of the Potomac River and east of the Mississippi), South (roughly, the secession states), Plains (between the Mississippi and the Rocky Mountains), and Mountain-Pacific (far-west mountain and coastal states).

## COMPARISONS OF ACHIEVEMENTS AND ASPIRATIONS

In the following paragraphs the results of selected questionnaire items are presented in verbal and tabular form. The brief commentary in each case attempts to point up key statistics and to compare the present state of affairs with the attitudes of educators, particularly those in engineering, toward economics.

### Curricular Strength of Economics

Although the need for a requirement in introductory economics in all engineering curricula has been recognized by many engineering educators, and recommended frequently in the past, only 76% of all schools are meeting this recommendation; about 10% are meeting it in part; and about 14% are not meeting it at all. It is doubtful whether the latter 20, or so, schools can justify

their omission of economics on the grounds that adding it would impair the quality of their technical training. Some of our most prominent engineering schools are private, and lie in the northeast or central United States -- a class of schools which, according to Table 1, most nearly meets recommended requirements in economics.

Despite a few notable exceptions, in those schools where general economics is either required or elective, Table 2 reveals that the average course length is only about four semester hours, instead of the six recommended to the Society for the Promotion of Engineering Education as far back as 1917.[4] Moreover, a glance at Table 3 indicates that, although the loss of economics education to campus military training does not pose a grave national problem, students in about 16% of schools do lose all or a part of their economics to R.O.T.C. Thus many additional student engineers get far fewer than the recommended hours.

Over-all, the curricular strength of economics for engineers appears to be stable or ebbing slightly. Two studies[5] made in 1950 reported both a higher proportion of schools requiring economics, and courses of longer average length than found in this study. Perhaps differences in the definition of a semester hour account for some of these discrepancies. A standard of

[4]Frank W. Skinner, Editor: Memoires and Addresses of Two Decades, by J.A.L. Waddell. Mack Printing Company, Easton, Pennsylvania, 1928. Pp. 546-7.

[5]See Albert J. Schweiger, "Survey of Economics and Related Subjects", Journal of Engineering Education, Vol. 41, No. 4, December 1950, pp. 231-4. Schweiger reported that 80% of engineering schools in a wide sample required economics of all students, for an average of 4.5 semester hours. See also Frederick A. Bradford, "Economics and the Study of Engineering", American Economic Review, Vol. XL, No. 5, December 1950, Part 2, pp. 180-1. On the basis of a sample of 95 schools, Bradford reported that 88.5% of schools required economics for an average of 4.5 semester hours. (The present study, based on returns from 145 schools, indicates 76% requiring economics of all students, and an average course length of about four semester hours.)

90 fifty-minute meetings for six semester hours was used in this study; also, the 1950 studies are based on smaller samples. But the notion that the position of economics has of late weakened somewhat is supported by the fact that at least three schools reported that they had recently dropped an economics requirement. At any rate, little evidence exists of a recent upsurge of interest, in economics in engineering schools, which might help close the gap between recommendations and accomplishments.

Aims

There has been, and is, little similarity in the thinking of engineers and of economics teachers, in respect to the aims of economics for engineers. General economics has been accepted in some engineering curricula for many years, but usually for reasons not thought valid by teachers of economics. However, the idea that general economics is not, exclusively, training for success in business or engineering, but education for rational citizenship as well, has been gradually gaining ground among engineers since World War I.

In practice, as pictured in Table 4, the hopes that economics will aid engineers professionally, and that it will make them more effective citizens, are being given equal weight. However, the additional aims of promoting an enriched intellectual life, providing training in logical thinking, and promoting more enlightened consumption, which are given little or no attention in the literature of engineering education, are receiving a substantial amount of emphasis in practice.

Special Courses and Methods?

Many engineering educators have expressed dissatisfaction over the years with the content of traditional economics, but, in practice, introductory content is still largely orthodox. Although the literature is replete with strong demands from engineers that special economics be given to engineering students, only about 22% of schools actually claim a course of special design (see Table 5). Furthermore, comments on the questionnaire indicated that those schools which do redesign economics for

engineers, are in disagreement on what kinds of changes are best. Many schools condense a traditional course, some "fortify" a standard course with mathematics, others stress business problems, and about a dozen schools offer "engineering economics" as a first course. However, since 124 schools emphasize Keynesian--post-Keynesian, neo-classical, and institutional economics, in that order (Table 6), since 70.4% of all schools combine theory and description into one course (Table 7), and since the questionnaire indicated that virtually all schools use standard textbooks, the traditional nature of the introductory course seems well established.

Some engineering opinion has persistently claimed that engineering students need special teaching methods and specially trained teachers of economics. However, the data in Table 8 show that in only 17% of all engineering schools do economics instructors seek, or have, special training prompted by their contacts with engineering students. Written remarks, in response to one questionnaire item, indicate that well under 10% of schools have recently experimented with method. Where special qualifications in economics teachers are sought, they follow rather closely the recommendations of the literature, i.e., that such teachers be older, more experienced men, with either practical business experience or a good background in mathematics and engineering.

According to Table 9, almost 50% of all schools teach economics to groups made up exclusively of engineers, but, since only 22% of schools claim special economics for engineers, one may conclude that, in a good many schools, philosophical rather than administrative barriers militate against special handling of engineers.

Integration

Although considerable attention has been given to designing integrated social science courses for engineers, economics is actually integrated into such courses in only about 3.8% of schools. Economics, in engineering curricula, has remained generally aloof from other social sciences. It is clear, from

Table 10, that economics is a separate course in three out of four schools, and part of a required sequence in the social sciences in only about one school in five. Furthermore, in about 84% of schools, there is no prerequisite to economics.

## SUMMARY

When one compares the aspirations for economics, as a prime ingredient in the engineering curriculum, with present achievements, a number of significant generalizations are possible. Briefly summarized, some of these conclusions are as follows:

(1) Most engineering (and arts) opinion seems to agree that all engineering students should be required to take introductory economics. Private schools in the Northeast-Central part of the country come closest to this ideal.

(2) There is general agreement that the requirement should be a six-semester-hour course. Engineers frequently regard this as a maximum; teachers of economics tend to regard it as a minimum. Again, schools in a Northeast-Central location come close to this ideal.

(3) The engineer in the past has most often emphasized a "professional" aim for teaching economics. This aim is most prominent in the large, public schools in the Mountain-Pacific area. Other aims, prominent in liberal arts thinking, are a little more popular in the older, smaller, private school.

(4) Many engineers prefer specially designed economics courses for their students; some engineers and most liberal arts people do not. Those favoring a special design would be most pleased by the economics in a small engineering school in the Plains states. Others would be more reassured by the economics in an older, southern engineering school of medium size.

(5) The engineer has frequently urged less abstraction in economics--less emphasis on theory and more on the institutional and historical approaches. His wish is best fulfilled in public schools in the Plains area. Economics people, who in the main would now favor post-Keynesian and neo-classical approaches, would be more at ease in private schools.

(6) In practice, no one class of schools conforms perfectly to a composite of engineering thinking about economics, yet if one were forced to pick the kind of school which offers economics in the manner closest to the engineering ideal, he would have to choose the large public school in the Plains or Mountain-Pacific states. The fact that this classification exhibits the least relative attention to economics, in terms of frequency of a requirement and length of course, suggests strongly that engineering educators still embrace economics only with reservation.

## IMPLICATIONS AND RECOMMENDATIONS

Although general economics has been in some technical curricula since 1827, and has steadily gained in importance, the thinking of engineering educators has often been at odds with that of economics educators. The engineering educator has frequently suggested that the economics teacher subordinate the intellectual and citizenship aims, which are given considerable priority in his thinking, to the need of engineers for more "practical" training in business and engineering economics. This, on balance, the economics teacher has been reluctant to do. That there is some lack of rapport between the engineer and the economics teacher is, indeed, not surprising. More disturbing, perhaps, is the fact that there seems at present no concerted effort to establish such rapport. There is, in other words, little evidence of a general awareness that <u>all three</u> aspects of economics--academic, business, and engineering--are <u>all</u> essential components of a first-rate technical education.

What can be done? Three suggestions immediately come to mind:

(1) The unanimous adoption of a specific set of objectives for an economics program for engineers, by an eminent group composed equally of academic, business, and engineering economists.

(2) General agreement by this group on the content essential to attain these objectives.

(3) Agreement on the minimum curricular time necessary for such a program.

The American Society for Engineering Education (with the cooperation of the American Economic Association) is well-equipped to undertake and carry out such a project. The Engineers' Council for Professional Development, by virtue of its accrediting power, is strategically placed to implement recommendations. Such a forthright attack on the problem of properly educating engineers in economics would certainly strengthen both the professional engineer, and ultimately, the American economy.

(Tables 1 through 10 follow.)

TABLE 1

EXTENT AND DISTRIBUTION OF AN INTRODUCTORY ECONOMICS REQUIREMENT IN AMERICAN ENGINEERING COLLEGES

(Expressed as Per Cent of Schools)

| TOTAL SAMPLE | | LOCATION | | | |
|---|---|---|---|---|---|
| Number of Schools - 145* | | NE-Cent. (73 Sch.) | South (28 Sch.) | Plains (22 Sch.) | Mt.-Pac. (22 Sch.) |
| Required | % | % | % | % | % |
| All | 76.0 | 80.9 | 78.5 | 68.2 | 63.7 |
| Some | 10.3 | 8.2 | 7.2 | 18.2 | 9.1 |
| None | 13.7 | 10.9 | 14.3 | 13.6 | 27.2 |

| AUSPICES | | | AGE | |
|---|---|---|---|---|
| | Public (85 Schools) | Private (60 Schools) | Old (38 Schools) | New (107 Schools) |
| Required | % | % | % | % |
| All | 69.4 | 85.0 | 76.3 | 75.7 |
| Some | 14.1 | 3.3 | 13.2 | 8.4 |
| None | 16.5 | 11.7 | 10.5 | 15.9 |

| SIZE | | | |
|---|---|---|---|
| | Large (34 Sch.) | Medium (73 Sch.) | Small (38 Sch.) |
| Required | % | % | % |
| All | 73.5 | 75.5 | 76.4 |
| Some | 11.8 | 10.9 | 7.9 |
| None | 14.7 | 13.6 | 15.7 |

*A return of 97.3 per cent.

TABLE 2

AVERAGE-SEMESTER-HOUR* LENGTH OF THE INTRODUCTORY ECONOMICS REQUIREMENT IN AMERICAN ENGINEERING COLLEGES

| TOTAL SAMPLE | SIZE | | | AUSPICES | |
|---|---|---|---|---|---|
| | Large | Medium | Small | Public | Private |
| 129 Schools** | 24 Sch. | 69 Sch. | 36 Sch. | 73 Sch. | 56 Sch. |
| 3.9 S-H | 3.5 | 4.0 | 4.0 | 3.8 | 4.1 |

| LOCATION | | | | AGE | |
|---|---|---|---|---|---|
| NE-Cent. | South | Plains | Mt.-Pac. | Old | New |
| 65 Sch. | 27 Sch. | 19 Sch. | 18 Sch. | 33 Sch. | 96 Sch. |
| 5.7 | 3.9 | 3.5 | 3.7 | 4.2 | 3.7 |

*Semester-hour is here defined as the equivalent of 15 class meetings of 50 minutes each.

**A return of 86.6 per cent.

TABLE 3

EXTENT AND DISTRIBUTION OF LOSS OF INTRODUCTORY ECONOMICS DUE TO R.O.T.C. IN AMERICAN ENGINEERING COLLEGES
(Expressed as Per Cent of Schools)

| TOTAL SAMPLE | | SIZE | | | AUSPICES | |
|---|---|---|---|---|---|---|
| | | Large | Medium | Small | Public | Private |
| 129 Schools* | | 30 Sch. | 65 Sch. | 34 Sch. | 72 Sch. | 57 Sch. |
| AFFECTED | % | % | % | % | % | % |
| All | 8.5 | 13.3 | 3.1 | 14.7 | 6.9 | 10.5 |
| None | 83.8 | 80.0 | 86.2 | 82.5 | 84.8 | 82.5 |
| Some | 7.7 | 6.7 | 10.7 | 2.8 | 8.3 | 7.0 |

| | AGE | | LOCATION | | | |
|---|---|---|---|---|---|---|
| | Old | New | NE-Cent. | South | Plains | Mt.-Pac. |
| | 33 Sch. | 96 Sch. | 64 Sch. | 26 Sch. | 19 Sch. | 20 Sch. |
| AFFECTED | % | % | % | % | % | % |
| All | 9.1 | 8.3 | 9.4 | 15.4 | | 5.0 |
| None | 78.8 | 85.4 | 87.5 | 73.1 | 79.0 | 90.0 |
| Some | 12.1 | 6.3 | 3.1 | 11.5 | 21.0 | 5.0 |

*A return of 86.6 per cent.

TABLE 4

EXPRESSED AIMS OF TEACHING INTRODUCTORY ECONOMICS
IN AMERICAN ENGINEERING SCHOOLS
(Expressed as Percentage Frequencies)

| | TOTAL SAMPLE | LOCATION | | | |
|---|---|---|---|---|---|
| | 133 Schools* Expressing | NE-Cent. | South | Plains | Mt.-Pac. |
| No. of Choices** | 348 | 192 | 57 | 49 | 50 |
| | % | % | % | % | % |
| Citizenship | 28.7 | 29.2 | 26.3 | 26.5 | 32.0 |
| Professional | 28.7 | 27.6 | 29.8 | 28.6 | 32.0 |
| Enrich Intellect | 21.5 | 22.9 | 24.6 | 18.4 | 16.0 |
| Logical Thinking | 13.0 | 13.5 | 10.5 | 18.4 | 8.0 |
| H'ds, H'scholds | 7.8 | 6.8 | 8.8 | 6.1 | 12.0 |
| Others | .3 | | | 2.0 | |

| | AUSPICES | | AGE | | SIZE | | |
|---|---|---|---|---|---|---|---|
| | Public | Private | Old | New | Large | Medium | Small |
| No. of Choices | 188 | 160 | 92 | 256 | 88 | 155 | 105 |
| | % | % | % | % | % | % | % |
| Citizenship | 27.7 | 30.0 | 27.2 | 29.3 | 27.3 | 28.4 | 30.5 |
| Professional | 30.3 | 27.9 | 27.2 | 29.3 | 29.6 | 29.6 | 26.6 |
| Enrich Intellect | 19.2 | 24.4 | 21.7 | 21.5 | 21.6 | 25.1 | 16.2 |
| Logical Thinking | 12.7 | 13.1 | 17.4 | 11.3 | 15.9 | 10.3 | 14.3 |
| H'ds, Households | 9.6 | 5.6 | 5.4 | 8.6 | 4.5 | 6.6 | 12.4 |
| Others | .5 | | 1.1 | | 1.1 | | |

* A return of 89.3 per cent.
** Many schools avowed more than one aim.

TABLE 5

EXTENT AND DISTRIBUTION OF INTRODUCTORY ECONOMICS COURSES
DESIGNED ESPECIALLY FOR ENGINEERS IN AMERICAN ENGINEERING COLLEGES
(Expressed as Per Cent of Schools)

| | TOTAL SAMPLE | LOCATION | | | |
|---|---|---|---|---|---|
| | | NE-Cent. | South | Plains | Mt.-Pac. |
| No. of Schools | 131* | 67 | 27 | 18 | 19 |
| Course: | % | % | % | % | % |
| Traditional | 78.0 | 76.2 | 88.9 | 61.2 | 84.2 |
| Special Design | 22.0 | 23.8 | 11.1 | 38.8 | 15.8 |

| | AUSPICES | | AGE | | SIZE | | |
|---|---|---|---|---|---|---|---|
| | Public | Private | Old | New | Large | Medium | Small |
| No. of Schools | 74 | 57 | 33 | 98 | 30 | 67 | 34 |
| Course: | % | % | % | % | % | % | % |
| Traditional | 77.0 | 79.0 | 84.9 | 75.5 | 73.4 | 83.6 | 70.6 |
| Special Design | 23.0 | 21.0 | 15.1 | 24.5 | 26.6 | 16.4 | 29.4 |

*A return of 88 per cent.

TABLE 6

EMPHASIS ON DIFFERENT APPROACHES TO INTRODUCTORY ECONOMICS IN AMERICAN ENGINEERING SCHOOLS

(Expressed as Percentage Frequencies)

| | TOTAL SAMPLE 124 Schools* Expressing | LOCATION | | | |
|---|---|---|---|---|---|
| | | NE-Cent. | South | Plains | Mt.-Pac. |
| No. of Choices** | 251 | 131 | 47 | 31 | 39 |
| | % | % | % | % | % |
| Classical | 8.0 | 8.2 | 6.4 | 3.2 | 12.8 |
| Neo-Classical | 25.9 | 25.4 | 29.8 | 25.8 | 23.1 |
| Keynes-post-Keynes | 32.6 | 34.3 | 31.9 | 25.8 | 33.3 |
| Institutional | 23.0 | 23.1 | 19.1 | 35.6 | 18.0 |
| Historical | 9.2 | 7.5 | 12.8 | 6.4 | 12.8 |
| Other | 1.3 | 1.5 | | 3.2 | |

| | AUSPICES | | AGE | | SIZE | | |
|---|---|---|---|---|---|---|---|
| | Public | Private | Old | New | Large | Medium | Small |
| No. of Choices | 151 | 100 | 66 | 185 | 64 | 125 | 62 |
| | % | % | % | % | % | % | % |
| Classical | 9.3 | 6.0 | 3.0 | 9.7 | 7.8 | 8.0 | 8.1 |
| Neo-Classical | 23.8 | 29.0 | 30.3 | 24.3 | 25.0 | 28.8 | 21.0 |
| Keynes-post-Keynes | 29.2 | 38.0 | 36.4 | 31.4 | 31.3 | 32.8 | 33.8 |
| Institutional | 24.5 | 21.0 | 24.3 | 22.7 | 23.4 | 20.8 | 27.4 |
| Historical | 11.9 | 5.0 | 4.5 | 10.8 | 7.8 | 9.6 | 9.7 |
| Other | 1.3 | 1.0 | 1.5 | 1.1 | 4.7 | | |

*A return of 83.3 per cent.

**Many schools indicated a combination of approaches.

TABLE 7

RELATIVE EMPHASIS ON THEORY AND DESCRIPTION
IN INTRODUCTORY ECONOMICS IN AMERICAN ENGINEERING SCHOOLS
(Expressed as Per Cent of Schools)

| | TOTAL SAMPLE | LOCATION | | | |
|---|---|---|---|---|---|
| | | NE-Cent. | South | Plains | Mt.-Pac. |
| No. of Schools | 132* | 70 | 25 | 17 | 20 |
| | % | % | % | % | % |
| Primarily Theory | 16.7 | 15.7 | 28.0 | 17.6 | 5.0 |
| Primarily Descr. | 9.9 | 10.0 | 4.0 | 11.7 | 15.0 |
| Combination | 70.4 | 71.5 | 68.0 | 64.8 | 75.0 |
| Other | 3.0 | 2.8 | | 5.9 | 5.0 |

| | AUSPICES | | AGE | | SIZE | | |
|---|---|---|---|---|---|---|---|
| | Public | Private | Old | New | Large | Medium | Small |
| No. of Schools | 75 | 57 | 34 | 98 | 31 | 65 | 36 |
| | % | % | % | % | % | % | % |
| Primarily Theory | 17.3 | 15.7 | 14.7 | 17.3 | 19.3 | 21.5 | 5.6 |
| Primarily Descr. | 10.7 | 8.8 | 5.9 | 11.2 | 9.7 | 10.8 | 8.3 |
| Combination | 68.0 | 73.7 | 73.5 | 69.5 | 67.8 | 66.2 | 80.5 |
| Other | 4.0 | 1.8 | 5.9 | 2.0 | 3.2 | 1.5 | 5.6 |

*A return of 88.5 per cent.

TABLE 8

EXTENT AND DISTRIBUTION OF SPECIAL TRAINING FOR THE
ECONOMICS TEACHERS OF STUDENTS IN AMERICAN ENGINEERING COLLEGES
(Expressed as Per Cent of Schools)

| | TOTAL SAMPLE | LOCATION | | | |
|---|---|---|---|---|---|
| | | NE-Cent. | South | Plains | Mt.-Pac. |
| No. of Schools | 73* | 38 | 15 | 11 | 11 |
| | % | % | % | % | % |
| Spec. Training | 17.3 | 21.0 | 13.3 | 18.2 | 9.1 |
| None | 82.7 | 79.0 | 86.7 | 81.8 | 90.9 |

| | AUSPICES | | AGE | | SIZE | | |
|---|---|---|---|---|---|---|---|
| | Public | Private | Old | New | Large | Medium | Small |
| No. of Schools | 38 | 37 | 15 | 60 | 22 | 38 | 15 |
| | % | % | % | % | % | % | % |
| Spec. Training | 10.5 | 24.3 | 33.3 | 13.3 | 13.6 | 18.4 | 20.0 |
| None | 89.5 | 75.7 | 66.7 | 86.7 | 86.4 | 81.6 | 80.0 |

*A return of 50.3 per cent.

TABLE 9

EXTENT AND DISTRIBUTION OF THE PRACTICE OF TEACHING INTRODUCTORY ECONOMICS TO GROUPS MADE UP EXCLUSIVELY OF ENGINEERS
(Expressed as Per Cent of Schools)

| | TOTAL SAMPLE | LOCATION | | | |
|---|---|---|---|---|---|
| | | NE-Cent. | South | Plains | Mt.-Pac. |
| No. of Schools | 129* | 70 | 24 | 16 | 19 |
| | % | % | % | % | % |
| Engineer Groups | 48.0 | 50.0 | 41.7 | 56.2 | 42.0 |
| Mixed Groups | 49.7 | 48.6 | 54.1 | 43.8 | 52.7 |
| Both Situations | 2.3 | 1.4 | 4.2 | | 5.3 |

| | AUSPICES | | AGE | | SIZE | | |
|---|---|---|---|---|---|---|---|
| | Public | Private | Old | New | Large | Medium | Small |
| No. of Schools | 73 | 56 | 33 | 98 | 27 | 66 | 36 |
| | % | % | % | % | % | % | % |
| Engineer Groups | 38.4 | 60.7 | 42.0 | 50.0 | 66.7 | 50.0 | 30.6 |
| Mixed Groups | 57.5 | 39.3 | 54.8 | 47.8 | 29.6 | 48.5 | 66.6 |
| Both Situations | 4.1 | | 3.2 | 2.2 | 3.7 | 1.5 | 2.8 |

*A return of 86.6 per cent.

TABLE 10

NATURE OF CURRICULA ARRANGEMENTS FOR INTRODUCTORY ECONOMICS IN AMERICAN ENGINEERING SCHOOLS
(Expressed as Per Cent of Schools)

| | TOTAL SAMPLE | LOCATION | | | |
|---|---|---|---|---|---|
| | | NE-Cent. | South | Plains | Mt.-Pac. |
| No. of Schools | 133* | 72 | 25 | 17 | 19 |
| | % | % | % | % | % |
| Separate Course | 75.2 | 77.7 | 60.0 | 64.7 | 94.7 |
| In Req. Sequence | 19.5 | 15.3 | 36.0 | 29.4 | 5.3 |
| Integrated | 3.8 | 5.6 | 4.0 | | |
| Other | 1.5 | 1.4 | | 5.9 | |

| | AUSPICES | | AGE | | SIZE | | |
|---|---|---|---|---|---|---|---|
| | Public | Private | Old | New | Large | Medium | Small |
| No. of Schools | 75 | 58 | 34 | 99 | 30 | 67 | 36 |
| | % | % | % | % | % | % | % |
| Separate Course | 74.6 | 75.8 | 85.3 | 71.8 | 83.4 | 73.1 | 72.2 |
| In Req. Seq. | 21.4 | 17.3 | 8.8 | 23.2 | 10.0 | 20.9 | 25.0 |
| Integrated | 2.6 | 5.2 | 5.9 | 3.0 | 3.3 | 4.5 | 2.8 |
| Other | 1.4 | 1.7 | | 2.0 | 3.3 | 1.5 | |

*A return of 89.3 per cent.

# NEWS AND NOTES

## NATIONAL ASSOCIATION OF BUSINESS ECONOMISTS

The National Association of Business Economists was organized in April 1959, to provide a forum for discussion of common problems by economists directly associated with business groups. According to the by-laws, the purpose is to "bring into focus the contribution which trained economists can make to business firms...provide an exchange of experience and ideas between the business community, schools of business, government, and economists in other fields, and to stimulate research into business policies affecting the national economy". Officers of the association are:

President - Adolph G. Abramson, SKF Industries, Inc.
Vice President - George Cline Smith, F. W. Dodge Corp.
Treasurer - Sidney E. Rolfe, Agora Development Corp.
Secretary - Charles B. Reeder, E. I. du Pont de Nemours & Co.

Membership is open to economists employed by business firms and trade associations, or who serve as consultants to business. An educator would qualify as consultant to a business organization.

Inquiries may be addressed to P.O. Box 6878, Philadelphia 32, Pennsylvania.

## LIBRARY ASSOCIATIONS PROJECT: Classification Schemes Sought

The Committee on Special Classifications of the Special Libraries Association and the Classification Committee of the Cataloging and Classification Section, Resources and Technical Services Division, American Library Association, are cooperating in a continuing project to develop and expand a Loan Collection of library classification schemes originally established by the Special Libraries Association. This collection covers all fields of science, law, medicine, technology, the social sciences and the humanities.

New libraries or libraries with special collections are constantly asking for classifications--in all areas of knowledge--and it is imperative that the Collection be kept up-to-date through the addition of new schemes or with modernized versions of existing classification schedules. Curators of special collections, special librarians, and those individuals who have developed special classification schemes for specific types of material or for special subjects are invited to contribute a copy of their work to the Collection. Classification schemes should be sent to Dr. Jesse H. Shera, Curator, SLA Loan Collection, School of Library Science, Western Reserve University, Cleveland 6, Ohio.

## 1959 SUMMER SYMPOSIUM PAPERS (ASEE-AIIE)

Copies of "Planning and Justifying Capital Expenditures" may be obtained by writing to The Engineering Economist. The price is $4 for members of ASEE and AIIE, for educators, schools and public libraries, and for those who attended the Symposium; and $10 for others.

## BACK ISSUES OF THE ENGINEERING ECONOMIST

The presently available back (single) issues of The Engineering Economist are Volume 4, Numbers 3 and 4, and Volume 5, Numbers 1 and 2; and (in limited supply) Volume 2, Number 2, and Volume 3, Numbers 1, 3 and 4.

A microfilm copy of the complete set of Volumes 1 through 4 may be obtained from University Microfilms, Inc., of 313 North First Street, Ann Arbor, Michigan. The price is $5.50 the set.

## INTEREST TABLES

The Interest Tables now being published serially in The Engineering Economist will be available at a later date in reprint form. The introductory material which appeared in Volume 5, Number 2 will appear in this reprint, together with the complete set of interest tables published earlier. See forthcoming issues for additional information.

# STEEL MAGAZINE SERIES ON DEPRECIATION REFORM

## A Review Article

By

David V. Heebink*
Stanford University

During March and April of 1959, STEEL magazine ran several articles on depreciation reform. This series was based, in part, on the results of a survey of 918 metalworking executives, who were asked to indicate what type of depreciation reform they preferred. It is interesting to note that the most popular scheme (the so-called "bracket system") polled 40% of the repondents' votes. Since the purpose of the series was to encourage industry to take political action towards the end of incorporating more liberal depreciation provisions into the Internal Revenue Code, industry's lack of agreement on a preferred reform was a major concern of the magazine's editors.

While the results of the survey are interesting, the purpose of this review is to describe and comment on the depreciation proposals that were covered in the STEEL articles, rather than to analyze the survey.

## THE BRACKET SYSTEM

The bracket system is not a method that can be used to write off an asset; it is, rather, a means of establishing the life over which a piece of property will be depreciated. Thus the bracket plan would replace Bulletin F as a means of determining allowable lives for income-tax purposes. This proposal consists of establishing ten to fifteen categories that are sufficiently broad, so that all classes of depreciable assets may be included. For

*The reviewer is indebted to Professor E. L. Grant for several valuable suggestions.

each of these categories, a maximum and a minimum allowable life would be determined. The taxpayer would be free to choose any figure within these limits. If he wished to use a life that was outside of the "bracket", the burden of proof would be on the taxpayer. STEEL claims that durable equipment would--for the most part--have a five- to ten-year bracket; the corresponding figures for buildings and fixed installations would be twenty to thirty years.

A form of the above plan is presently in use in Canada. The chief difference seems to be that the Canadian scheme does not set an upper limit on depreciable life. According to STEEL, an upper limit is desirable in order to "discourage wide variations in write-offs".[1] The magazine claims that the primary advantage of this plan is: "Brackets will allow depreciable lives short enough so that the prospect for the depreciation write-off does not stand in the way of a necessary replacement or addition."[2] The bracket plan seems to have some influential supporters, such as the United States Chamber of Commerce and the National Machine Tool Builders Association.

STEEL regards the bracket system as a radical departure from current depreciation practice. In fact it expresses considerable doubt as to whether Congress would accept the plan. This reviewer will make no attempt to anticipate the political fate of the proposal; however, a closer look at the ways in which it differs from current practice seems to be in order. In contrast to Bulletin F, the bracket system establishes a finite upper limit for depreciable life. A more important difference is that STEEL is apparently advocating a reduction in allowable lives as part of the plan. How desirable are these changes?

Placing an upper limit on allowable life hardly seems worth the effort. At the present time a taxpayer is free to use a longer life than that set forth in Bulletin F; however, the lower limit is the important one, because the vast majority of tax-

[1] STEEL, March 16, 1959, p. 67.
[2] Ibid.

payers use as short a life as the law allows. Moreover, the reviewer does not share STEEL'S concern about "wide variations in write-off". Unless variations are to be completely eliminated for administrative reasons, why should the range over which they occur be restricted at all? The advantage to the taxpayer and the Internal Revenue Service is not apparent.

Reducing allowable lives is the radical aspect of the bracket system. This is clearly one way to increase the rate at which assets are written off. However, even though faster write-off may be desirable, it does not necessarily follow that the bracket plan is the best way to accomplish this objective. If the plan were adopted, the document that replaced Bulletin F would no doubt be easier to prepare and use. However, permissible brackets for various types of property likely would be determined by administrative fiat, and political pressures could influence the process. In time, allowable lives might well become completely distorted. The reviewer is of the opinion that there are other means of obtaining fast write-off which are less susceptible to lobbying by special-interest groups.

## THE MAPI PLAN

The depreciation reforms proposed by George Terborgh, of the Machinery and Allied Products Institute, received 37.7% of the votes in the STEEL poll. This approach is probably the least novel of any covered in the magazine's series. The basic elements of present depreciation practice, such as useful-life and declining-balance depreciation, are retained; the objective of the MAPI plan is to allow for the effects of monetary inflation by increasing the rate at which assets may be written off. STEEL indicates that MAPI has endorsed two alternate schemes for doing this. The first is triple-declining-balance depreciation, i.e., writing off assets, according to the usual declining-balance method, at a maximum permissible rate that is three times the allowable straight-line rate. The second approach is similar to that used by the British. The tax law in England allows a special

write-off equal to 30% of the first cost of equipment (15% in the case of buildings) during the first year of an asset's life.

STEEL implies that the added depreciation write-off, made possible by either of these proposals, would largely eliminate the inadequacy of current depreciation charges in this country. The magazine claims that the annual deficiency in these charges is now between $6 billion and $8 billion dollars. This situation could be corrected immediately, if the British system were put into effect, or--if triple-declining-balance depreciation were used--the deficiency would be largely offset in five years' time, according to STEEL.

The MAPI declining-balance plan has the advantage of not being radically different. Congress is more likely to accept an alteration of an existing practice than a brand new one. Neither MAPI alternative would be any more difficult to administer than depreciation methods which are now in effect. The only remaining issue, in this instance, is the question of whether more rapid write-off is in fact desirable. More will be said about this matter below.

## REINVESTMENT DEPRECIATION

The third most popular depreciation reform in the poll (12.7% of the votes) is that proposed by an accountant, Mr. M. E. Peloubet. This method attempts to counteract the effects of inflation in a manner that, according to STEEL, is analogous to the LIFO inventory scheme. When an asset is retired, the difference between its original cost, and the current cost of an equivalent piece of property, is determined. The reinvestment-depreciation method would allow all or any part of this difference as a tax-deductible expense "... to the extent that an equivalent investment is made within two years of retirement".[3]

This approach involves the use of a cost index. STEEL gives

[3] STEEL, March 30, 1959, p. 54.

the following example of reinvestment depreciation: "Suppose that in 1959 a taxpayer dismantles a machine purchased in 1939 for $50,000 and fully depreciated since then. Assume that the cost index shows an increase in price levels of 130% from 1939 to 1959. The taxpayer may elect to deduct in 1959 the cost of the property purchased that year to the extent that its cost exceeds $50,000 but not more than $115,000 (230% times $50,000). The maximum deduction is $65,000, or the equivalent of the 130% cost increase. The remaining $50,000 can be depreciated at normal rates.

"Another example: Suppose that in the previous case new investment is only $60,000 in 1959 but that additional investment amounting to $200,000 is made in 1960. The taxpayer would take reinvestment depreciation deductions of $10,000 in 1959 and $55,000 in 1960--the total of $65,000 in deductions for the two years requiring the 130% index increase multiplied by the $50,000 original cost."[4]

Assuming for the moment that depreciation schemes should, in some manner, allow for the effects of monetary inflation, reinvestment depreciation is one possible way of doing this. However, from an administrative standpoint, it would create some difficult problems. First, how does one determine what is an "equivalent investment"? Unless the Internal Revenue Service could accept <u>any</u> investment in plant and equipment as an equivalent investment, this aspect of the proposal would be very troublesome. It seems doubtful that such a broad interpretation of equivalent investment would be acceptable to Congress.

A second problem is that of determining the appropriate cost index to use for various types of plant and equipment. Inflation has not had an identical effect on all classes of assets; the prices of some have risen faster than others. Therefore, an over-all index hardly seems appropriate. A lot of time and effort

[4]STEEL, March 30, 1959, p. 54.

would have to be spent in order to establish and perpetuate a sufficient number of indices to do the job in an equitable fashion. In addition, there is the problem of eliminating the effects of technological change on the price level of rapidly developing types of equipment. If the price of a certain kind of machinery doubles in five years, and substantial technical improvements are made as well, how much of the 100% increase should be attributed to inflation and how much to technical advances?

Another disadvantage of reinvestment depreciation is that it is not equitable. It protects investments in depreciable property only. This is obviously unfair to the owner of non-depreciable assets (e.g., government bonds). Also, this scheme discriminates against the new business venture. Compared to an older, mature competitor, a new company would--in effect--have to pay more for a given piece of new equipment, because its re-investment deductions would generally be smaller or non-existent.

A point in favor of reinvestment depreciation is that it would provide management with a tax incentive for getting rid of much old, obsolete equipment. However, this plan is not the only way to avoid technological stagnation, and it is not a foregone conclusion that depreciation ought to compensate for the effects of inflation in the first place.

## INITIAL WRITE-OFF METHOD

The least popular depreciation reform covered in the STEEL poll also requires the least change in the tax law. Although it received only 1.8% of the votes, initial write-off is already part of the law; but its use is very restricted. Section 179 of the Code allows the taxpayer to write off 20% of the cost of a depreciable asset during the first year of its life, if the first cost is not more than $10,000, and if the asset has a useful life of at least six years. The proposed reform is to remove the dollar limit from the law. This is essentially a modification of the British alternative suggested by MAPI. The initial write-off

reform would of course be a less drastic change (20% instead of 30%), and equally feasible from an administrative point of view. It is interesting to note that Mr. Peloubet has indicated that, in his opinion, removing the $10,000 limit would have about the same end-result as his reinvestment-depreciation proposal. Assuming this is true, initial write-off appears to be the preferable method, because of its administrative simplicity.

## DEPRECIATION AND INFLATION

STEEL'S case for more rapid write-off is based entirely on the argument that inflation, combined with existing depreciation methods, is causing serious capital erosion. During periods when income taxes are high, this is certainly a problem; however, one ought to keep in mind that--strictly speaking--depreciation is the allocation, over time, of a prepaid expense. As such, depreciation is not expected to provide protection against changes in the price level. The risk of capital erosion is one of many confronting the businessman; and when a person advocates eliminating this risk, by means of special depreciation measures, he is in fact suggesting that depreciation be used to provide a form of business insurance, as well as a means of allocating a fixed expense.

This reviewer takes the position that reducing the risk of capital erosion, by means of more rapid write-off, is quite desirable. But including insurance against capital erosion, as part of an allowable depreciation scheme, is not only inconsistent with traditional concepts of depreciation, it is misleading as well. If insurance against capital erosion is to be provided by Congress, it should not be disguised as a depreciation reform.

## CONCLUSION

For reasons indicated above, the bracket plan and reinvestment depreciation are unsatisfactory. The former is a politically vulnerable substitute for Bulletin F; and the latter, in addition to having serious administrative drawbacks, goes farther in

dealing with the problem of inflation than a depreciation reform really should.

This leaves the triple-declining-balance and initial write-off (either 20% or 30%) methods. Both of these reforms reduce the risk of capital erosion by allowing for faster write-off, and neither would cause serious administrative problems. Thus, neither one can be automatically eliminated. In the opinion of the reviewer the triple-declining-balance scheme is preferable, because it is more straightforward. It makes possible a large depreciation charge during the first year of an asset's life, without necessitating another "patch" in the Internal Revenue Code. On the other hand, an initial write-off measure that was widely used would have the effect of compounding depreciation schemes, i.e., property would be written off according to one set of rules during the first year of its life, and according to another set thereafter.

* * *

# "LONG-RANGE PLANNING FOR MANAGEMENT"*

A Review

By

W. D. McEachron
Standard Oil Company (Indiana)

This book is a collection of papers originally written, for the most part, during the period from 1954 through June 1957, and gathered from a variety of sources. One-third of the contributed material comes from sections of other books, and another third from Harvard publications, primarily the Harvard Business Review. The remaining third is about equally divided among other professional societies and corporate manuals.

Some thirty-five authors have contributed their individual writings to this book, and each such writing constitutes a separate chapter. Editor Ewing has loosely organized these separate chapters into six sections, dealing in turn with principles, organization, planning steps, special problems, limitations, and strategy. Each such section is marked by an introductory chapter, in which Mr. Ewing briefs the reader on the background of the several contributors and points up the conclusions and contrasts that can be drawn from the material to follow. After this, the reader is on his own.

Because of the different approaches and writing styles of the many contributors, the great variation in chapter length (from two to thirty-two pages), and above all the overlapping and contradictory nature of the content, this is not an easy book to read through from cover to cover. Mr. Ewing feels that these contradictions are inherent in the art of long-range planning, at the present stage of its development. They certainly are characteristic of the literature on the subject.

*DAVID W. EWING, Editor: Harper & Brothers, New York, 1958. 489 pp. $6.50.

For the serious reader who is willing to dig, however, the book has plenty to offer in terms of general guidance and a miscellany of ideas, which should help to give considerable insight into problems in the long-range-planning field. Case histories in planning are given for about ten companies, and, in a few instances, in considerable detail. Chapter 12, dealing with Lockheed Aircraft Corporation, is outstanding in this regard.

Other chapters deal more generally with long-range planning, as it might apply to almost any corporation. The approach,in these chapters, ranges from broad generalizations about organization and planning principles to specific check-lists regarding methodology and problem areas. The contributions of Wrapp and Drucker are good examples in the former category, while the chapters by Payne and Powlison are both of considerable value in the more specific areas.

Still other chapters are so remote from planning that their inclusion is rather surprising. Notable examples include a fairly lengthy story on business espionage, and one on the obstacles encountered in creating a market for aluminum sleeve bearings. Perhaps these help give broad perspective to the subject of business planning; they certainly are different.

Most of the material is management-oriented, with little attention given to the specific problems in the technique of planning itself. Much of the writing is cast in terms of easy generalities that are perfectly obvious on hindsight, but may not be so apparent until the reader sees them in print and organized in a logical framework. It is this logical organization that gives utility to this type of treatment.

The nature of the book does not lend itself to any complete summary of its content, but a few remarks along this line may be in order. Apparently, the way in which long-range planning is organized is the factor most critical to its success. Most of the authors--but not all--feel that it is more nearly a line than a staff function, and all agree that it must be spearheaded by top management. Much emphasis is given to the need for defining corporate objectives and planning goals, in advance; and this is

not nearly as easy as it seems. Personnel planning comes in for considerable attention, at a number of points, and plans in this area may need to range as far ahead as twenty years. For facilities and operations planning, however, the norm seems closer to five years.

How much useful literature has been omitted, that should have been included, is hard to say. Prior to the cutoff date of June 1957, the subject of long-range planning was only beginning to become popular, so that the list of potential material may not have been very long. The need to secure permission for reprinting may have compounded the problem. Even so, a broader coverage of professional journals, beyond the Harvard Business Review, would have been desirable. Along this line, an example that comes to mind is Ewing W. Reilly's excellent article in the December 1955 issue of Advanced Management, entitled "Planning the Strategy of the Business".

All in all, LONG-RANGE PLANNING FOR MANAGEMENT provides a useful frame of reference to the planning literature of the period it covers. While it will certainly prove disappointing to those who want specific ready-made answers, it should be very helpful to the newcomer who wishes to survey the field, or to those experienced in planning who wish to test their own ideas against the observations of others.

* * *

# "APPLICATION OF LINEAR PROGRAMMING TO FINANCIAL BUDGETING AND THE COSTING OF FUNDS"*

A Review

By

William T. Morris
The Ohio State University

This is a classic study, by the "Old Masters", of the method-centered approach to management problems. It demonstrates once again the extraordinary power of the linear-programming framework, in dealing with complex business-decision problems. Students of the capital-budgeting process will find it filled with insights, and highly suggestive. It may eventually become the established basis for a revised conception of the interaction between financial planning and the economic analysis of engineering projects.

The stated purpose of the study is to explore ways in which linear programming might be used to allocate funds within a firm. It reveals the possibility of capturing, within a single conceptual model, the following interrelated questions:

1. If we have a firm with a given collection of facilities, what is the best (most profitable) program of operation for these facilities?
2. If we change the facilities as the result of the introduction of new engineering projects, how will this influence our operating program, as determined above? What will be the maximum profit increase if the new facilities are used in the most profitable way? This increase provides a measure of the yield or return on the engineering projects.
3. What is the cost of funds invested and optimally employed by the firm?

It is possible, within a single formulation, to deal with

*A. CHARNES, W. W. COOPER, and M. H. MILLER: The Journal of Business, Vol. XXXII, No. 1, January 1959, pp. 20-46.

the sometimes complicated interactions between financial and operating decisions.

The study goes on to illustrate this possibility, using a warehouse example, in which a firm buys and sells a commodity under certain liquidity restrictions. The number of insights to be obtained is surprising. To name a few, it is, for example, possible to obtain the rates of return on funds invested optimally in various ways, the costs of all sorts of financial arrangements involving borrowing and lending, the value of changing the liquidity requirements of the firm, and the costs of utilizing trade credit. The important point is that these results are obtained from a consideration of the simultaneous problems of financial and operating policy.

The reader will find it possible to translate this example into one involving equipment and manufacturing, with analogous results. As in all such explicit analysis of large segments of systems, one is likely to be a little dismayed by the amount of data which would be required to actually make an application (to say nothing of the computer time required to handle the data). However, it is well to keep two notions in mind. The development of such a powerful approach may provide the incentive to set in motion the processes of routinely accumulating the required information. Meanwhile, a study of the model itself can increase one's "feel" for the problem, without the necessity for precise data. This latter use is taken by many to yield the real payoff from engineering analysis.

Although the article is clearly and exhaustively written, the reader may feel more at ease if he first refreshes his memory in the areas of the general structure of linear-programming models, the dual problem, and the simplex or some other computational algorithm.

* * *

# PART II
## AN EXTENSION OF AVAILABLE INTEREST TABLES

By

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The interest tables which are presented on the following pages are a continuation of those which were published in Volume 5, Number 2, of The Engineering Economist. Part I included, in addition to tables, introductory material, examples of the uses of the tables, and derivations of the equations on which some of the less commonly found tables are based. The calculations for all of the tables were performed in the Syracuse University Computing Center, using an IBM 650 machine, and the work was partially supported by a grant from the National Science Foundation.

Other tables will be published in Volume 5, Number 4. At a later date, the complete series will be published in reprint form, together with the introduction. Announcements concerning this reprint will appear in the journal.

* * *

## Capital Recovery Factor - Annual Compounding

| n | 5% | 10% | 15% | 20% | 25% | 30% |
|---|---|---|---|---|---|---|
| 1 | 1.0500 | 1.1000 | 1.1500 | 1.2000 | 1.2500 | 1.3000 |
| 2 | 0.5378 | 0.5762 | 0.6151 | 0.6545 | 0.6944 | 0.7348 |
| 3 | 0.3672 | 0.4021 | 0.4380 | 0.4747 | 0.5123 | 0.5506 |
| 4 | 0.2820 | 0.3155 | 0.3503 | 0.3863 | 0.4234 | 0.4616 |
| 5 | 0.2310 | 0.2638 | 0.2983 | 0.3344 | 0.3718 | 0.4106 |
| 6 | 0.1970 | 0.2296 | 0.2642 | 0.3007 | 0.3388 | 0.3784 |
| 7 | 0.1728 | 0.2054 | 0.2404 | 0.2774 | 0.3163 | 0.3569 |
| 8 | 0.1547 | 0.1874 | 0.2228 | 0.2606 | 0.3004 | 0.3419 |
| 9 | 0.1407 | 0.1736 | 0.2096 | 0.2481 | 0.2888 | 0.3312 |
| 10 | 0.1295 | 0.1628 | 0.1992 | 0.2385 | 0.2801 | 0.3235 |
| 11 | 0.1204 | 0.1540 | 0.1911 | 0.2311 | 0.2735 | 0.3177 |
| 12 | 0.1128 | 0.1468 | 0.1845 | 0.2253 | 0.2684 | 0.3135 |
| 13 | 0.1065 | 0.1408 | 0.1791 | 0.2206 | 0.2645 | 0.3102 |
| 14 | 0.1010 | 0.1358 | 0.1747 | 0.2169 | 0.2615 | 0.3078 |
| 15 | 0.0963 | 0.1315 | 0.1710 | 0.2139 | 0.2591 | 0.3060 |
| 16 | 0.0923 | 0.1278 | 0.1680 | 0.2114 | 0.2572 | 0.3046 |
| 17 | 0.0887 | 0.1247 | 0.1654 | 0.2094 | 0.2558 | 0.3035 |
| 18 | 0.0856 | 0.1219 | 0.1632 | 0.2078 | 0.2546 | 0.3027 |
| 19 | 0.0828 | 0.1196 | 0.1613 | 0.2065 | 0.2537 | 0.3021 |
| 20 | 0.0802 | 0.1175 | 0.1598 | 0.2054 | 0.2529 | 0.3016 |
| 25 | 0.0710 | 0.1102 | 0.1547 | 0.2021 | 0.2509 | 0.3004 |
| 30 | 0.0650 | 0.1061 | 0.1523 | 0.2008 | 0.2503 | 0.3001 |
| 35 | 0.0611 | 0.1037 | 0.1511 | 0.2003 | 0.2501 | 0.3000 |
| 40 | 0.0583 | 0.1023 | 0.1505 | 0.2001 | 0.2500 | |
| 45 | 0.0563 | 0.1014 | 0.1503 | 0.2001 | | |
| 50 | 0.0548 | 0.1009 | 0.1501 | 0.2000 | | |

## Capital Recovery Factor - Annual Compounding

| n | 35% | 40% | 45% | 50% | 55% | 60% | 65% |
|---|---|---|---|---|---|---|---|
| 1 | 1.3500 | 1.4000 | 1.4500 | 1.5000 | 1.5500 | 1.6000 | 1.6500 |
| 2 | 0.7755 | 0.8167 | 0.8582 | 0.9000 | 0.9421 | 0.9846 | 1.0273 |
| 3 | 0.5897 | 0.6294 | 0.6697 | 0.7105 | 0.7519 | 0.7938 | 0.8361 |
| 4 | 0.5008 | 0.5408 | 0.5816 | 0.6231 | 0.6652 | 0.7080 | 0.7514 |
| 5 | 0.4505 | 0.4914 | 0.5332 | 0.5758 | 0.6192 | 0.6632 | 0.7079 |
| 6 | 0.4193 | 0.4613 | 0.5043 | 0.5481 | 0.5927 | 0.6380 | 0.6839 |
| 7 | 0.3988 | 0.4419 | 0.4861 | 0.5311 | 0.5768 | 0.6232 | 0.6701 |
| 8 | 0.3849 | 0.4291 | 0.4743 | 0.5203 | 0.5670 | 0.6143 | 0.6621 |
| 9 | 0.3752 | 0.4203 | 0.4665 | 0.5134 | 0.5609 | 0.6089 | 0.6573 |
| 10 | 0.3683 | 0.4143 | 0.4612 | 0.5088 | 0.5570 | 0.6055 | 0.6544 |
| 11 | 0.3634 | 0.4101 | 0.4577 | 0.5058 | 0.5545 | 0.6034 | 0.6526 |
| 12 | 0.3598 | 0.4071 | 0.4553 | 0.5039 | 0.5529 | 0.6021 | 0.6516 |
| 13 | 0.3572 | 0.4051 | 0.4536 | 0.5026 | 0.5519 | 0.6013 | 0.6510 |
| 14 | 0.3553 | 0.4036 | 0.4525 | 0.5017 | 0.5512 | 0.6008 | 0.6506 |
| 15 | 0.3539 | 0.4026 | 0.4517 | 0.5011 | 0.5508 | 0.6005 | 0.6504 |
| 16 | 0.3529 | 0.4018 | 0.4512 | 0.5008 | 0.5505 | 0.6003 | 0.6502 |
| 17 | 0.3521 | 0.4013 | 0.4508 | 0.5005 | 0.5503 | 0.6002 | 0.6501 |
| 18 | 0.3516 | 0.4009 | 0.4506 | 0.5003 | 0.5502 | 0.6001 | 0.6501 |
| 19 | 0.3512 | 0.4007 | 0.4504 | 0.5002 | 0.5501 | 0.6001 | 0.6500 |
| 20 | 0.3509 | 0.4005 | 0.4503 | 0.5002 | 0.5501 | 0.6000 | |
| 25 | 0.3502 | 0.4001 | 0.4500 | 0.5000 | 0.5500 | | |
| 30 | 0.3500 | 0.4000 | | | | | |
| 35 | | | | | | | |
| 40 | | | | | | | |
| 45 | | | | | | | |
| 50 | | | | | | | |

## Capital Recovery Factor - Annual Compounding

| n | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
|---|---|---|---|---|---|---|---|
| 1 | 1.7000 | 1.7500 | 1.8000 | 1.8500 | 1.9000 | 1.9500 | 2.0000 |
| 2 | 1.0704 | 1.1136 | 1.1571 | 1.2009 | 1.2448 | 1.2890 | 1.3333 |
| 3 | 0.8789 | 0.9220 | 0.9656 | 1.0094 | 1.0536 | 1.0981 | 1.1429 |
| 4 | 0.7952 | 0.8395 | 0.8842 | 0.9293 | 0.9748 | 1.0206 | 1.0667 |
| 5 | 0.7530 | 0.7987 | 0.8447 | 0.8911 | 0.9379 | 0.9849 | 1.0323 |
| 6 | 0.7303 | 0.7771 | 0.8242 | 0.8717 | 0.9195 | 0.9676 | 1.0159 |
| 7 | 0.7175 | 0.7652 | 0.8133 | 0.8616 | 0.9102 | 0.9589 | 1.0079 |
| 8 | 0.7102 | 0.7586 | 0.8073 | 0.8562 | 0.9053 | 0.9546 | 1.0039 |
| 9 | 0.7060 | 0.7549 | 0.8041 | 0.8534 | 0.9028 | 0.9523 | 1.0020 |
| 10 | 0.7035 | 0.7528 | 0.8022 | 0.8518 | 0.9015 | 0.9512 | 1.0010 |
| 11 | 0.7020 | 0.7516 | 0.8012 | 0.8510 | 0.9008 | 0.9506 | 1.0005 |
| 12 | 0.7012 | 0.7509 | 0.8007 | 0.8505 | 0.9004 | 0.9503 | 1.0002 |
| 13 | 0.7007 | 0.7505 | 0.8004 | 0.8503 | 0.9002 | 0.9502 | 1.0001 |
| 14 | 0.7004 | 0.7503 | 0.8002 | 0.8502 | 0.9001 | 0.9501 | 1.0000 |
| 15 | 0.7002 | 0.7502 | 0.8001 | 0.8501 | 0.9001 | 0.9500 | |
| 16 | 0.7001 | 0.7501 | 0.8001 | 0.8500 | 0.9000 | | |
| 17 | 0.7001 | 0.7501 | 0.8000 | | | | |
| 18 | 0.7000 | | | | | | |
| 19 | | | | | | | |
| 20 | | | | | | | |
| 25 | | | | | | | |
| 30 | | | | | | | |
| 35 | | | | | | | |
| 40 | | | | | | | |
| 45 | | | | | | | |
| 50 | | | | | | | |

## Gradient Factor - Uniform Equivalent - Annual Compounding

| n | 2% | 4% | 6% | 8% | 10% | 12% |
|---|---|---|---|---|---|---|
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.4951 | 0.4902 | 0.4854 | 0.4808 | 0.4762 | 0.4717 |
| 3 | 0.9867 | 0.9739 | 0.9612 | 0.9487 | 0.9366 | 0.9246 |
| 4 | 1.475 | 1.451 | 1.427 | 1.404 | 1.381 | 1.359 |
| 5 | 1.960 | 1.922 | 1.883 | 1.846 | 1.810 | 1.775 |
| 6 | 2.442 | 2.386 | 2.330 | 2.276 | 2.224 | 2.172 |
| 7 | 2.920 | 2.843 | 2.768 | 2.694 | 2.622 | 2.551 |
| 8 | 3.396 | 3.294 | 3.195 | 3.099 | 3.004 | 2.913 |
| 9 | 3.868 | 3.739 | 3.613 | 3.491 | 3.372 | 3.257 |
| 10 | 4.337 | 4.177 | 4.022 | 3.871 | 3.725 | 3.585 |
| 11 | 4.802 | 4.609 | 4.421 | 4.239 | 4.064 | 3.895 |
| 12 | 5.264 | 5.034 | 4.811 | 4.596 | 4.388 | 4.190 |
| 13 | 5.723 | 5.453 | 5.192 | 4.940 | 4.699 | 4.468 |
| 14 | 6.179 | 5.866 | 5.564 | 5.273 | 4.996 | 4.731 |
| 15 | 6.631 | 6.272 | 5.926 | 5.594 | 5.279 | 4.980 |
| 16 | 7.080 | 6.672 | 6.279 | 5.904 | 5.549 | 5.215 |
| 17 | 7.526 | 7.066 | 6.623 | 6.204 | 5.807 | 5.435 |
| 18 | 7.968 | 7.453 | 6.960 | 6.492 | 6.053 | 5.643 |
| 19 | 8.407 | 7.834 | 7.287 | 6.770 | 6.286 | 5.838 |
| 20 | 8.843 | 8.209 | 7.605 | 7.037 | 6.508 | 6.020 |
| 21 | 9.275 | 8.578 | 7.915 | 7.294 | 6.719 | 6.191 |
| 22 | 9.705 | 8.941 | 8.217 | 7.541 | 6.919 | 6.351 |
| 23 | 10.13 | 9.297 | 8.510 | 7.779 | 7.108 | 6.501 |
| 24 | 10.55 | 9.648 | 8.795 | 8.007 | 7.288 | 6.640 |
| 25 | 10.97 | 9.992 | 9.072 | 8.225 | 7.458 | 6.771 |
| 30 | 13.02 | 11.62 | 10.34 | 9.190 | 8.176 | 7.297 |
| 35 | 15.00 | 13.12 | 11.43 | 9.961 | 8.709 | 7.658 |
| 40 | 16.89 | 14.48 | 12.36 | 10.57 | 9.096 | 7.899 |
| 45 | 18.70 | 15.70 | 13.14 | 11.04 | 9.374 | 8.057 |
| 50 | 20.44 | 16.81 | 13.80 | 11.41 | 9.570 | 8.160 |

## Gradient Factor - Uniform Equivalent - Annual Compounding

| n | 14% | 16% | 18% | 20% | 25% | 30% |
|---|---|---|---|---|---|---|
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.4673 | 0.4630 | .4587 | .4545 | 0.4444 | 0.4347 |
| 3 | 0.9129 | .9014 | .8901 | 0.8791 | 0.8525 | .8271 |
| 4 | 1.337 | 1.316 | 1.295 | 1.274 | 1.225 | 1.178 |
| 5 | 1.740 | 1.706 | 1.673 | 1.640 | 1.563 | 1.490 |
| 6 | 2.122 | 2.073 | 2.025 | 1.979 | 1.868 | 1.765 |
| 7 | 2.483 | 2.417 | 2.353 | 2.290 | 2.142 | 2.006 |
| 8 | 2.825 | 2.739 | 2.656 | 2.576 | 2.387 | 2.216 |
| 9 | 3.146 | 3.039 | 2.936 | 2.836 | 2.605 | 2.396 |
| 10 | 3.449 | 3.319 | 3.194 | 3.074 | 2.797 | 2.551 |
| 11 | 3.733 | 3.578 | 3.430 | 3.289 | 2.966 | 2.683 |
| 12 | 4.000 | 3.819 | 3.647 | 3.484 | 3.115 | 2.795 |
| 13 | 4.249 | 4.041 | 3.845 | 3.660 | 3.244 | 2.889 |
| 14 | 4.482 | 4.246 | 4.025 | 3.817 | 3.356 | 2.969 |
| 15 | 4.699 | 4.435 | 4.189 | 3.959 | 3.453 | 3.034 |
| 16 | 4.901 | 4.609 | 4.337 | 4.085 | 3.537 | 3.089 |
| 17 | 5.089 | 4.768 | 4.471 | 4.198 | 3.608 | 3.135 |
| 18 | 5.263 | 4.913 | 4.592 | 4.298 | 3.670 | 3.172 |
| 19 | 5.424 | 5.046 | 4.700 | 4.386 | 3.722 | 3.202 |
| 20 | 5.573 | 5.167 | 4.798 | 4.464 | 3.767 | 3.228 |
| 21 | 5.711 | 5.277 | 4.885 | 4.533 | 3.805 | 3.248 |
| 22 | 5.838 | 5.377 | 4.963 | 4.594 | 3.836 | 3.265 |
| 23 | 5.955 | 5.467 | 5.033 | 4.647 | 3.863 | 3.278 |
| 24 | 6.063 | 5.549 | 5.095 | 4.694 | 3.886 | 3.289 |
| 25 | 6.161 | 5.623 | 5.150 | 4.735 | 3.905 | 3.299 |
| 30 | 6.542 | 5.896 | 5.345 | 4.873 | 3.963 | 3.322 |
| 35 | 6.782 | 6.055 | 5.449 | 4.941 | 3.986 | 3.330 |
| 40 | 6.930 | 6.144 | 5.503 | 4.973 | 3.995 | 3.332 |
| 45 | 7.019 | 6.193 | 5.529 | 4.988 | 3.999 | 3.333 |
| 50 | 7.071 | 6.220 | 5.542 | 4.995 | 3.998 | 3.333 |

## Gradient Factor - Uniform Equivalent - Annual Compounding

| n | 35% | 40% | 45% | 50% | 55% | 60% | 65% |
|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.4255 | 0.4167 | 0.4082 | 0.4000 | 0.3922 | 0.3846 | 0.3774 |
| 3 | 0.8029 | 0.7798 | 0.7578 | 0.7368 | 0.7168 | 0.6977 | 0.6794 |
| 4 | 1.134 | 1.092 | 1.053 | 1.015 | 0.9800 | 0.9464 | 0.9146 |
| 5 | 1.422 | 1.358 | 1.298 | 1.242 | 1.189 | 1.140 | 1.093 |
| 6 | 1.670 | 1.581 | 1.499 | 1.423 | 1.352 | 1.286 | 1.226 |
| 7 | 1.881 | 1.766 | 1.661 | 1.565 | 1.477 | 1.400 | 1.322 |
| 8 | 2.060 | 1.919 | 1.791 | 1.675 | 1.570 | 1.476 | 1.390 |
| 9 | 2.209 | 2.042 | 1.893 | 1.760 | 1.640 | 1.534 | 1.438 |
| 10 | 2.334 | 2.142 | 1.973 | 1.824 | 1.692 | 1.575 | 1.471 |
| 11 | 2.436 | 2.221 | 2.034 | 1.871 | 1.729 | 1.604 | 1.494 |
| 12 | 2.520 | 2.285 | 2.082 | 1.907 | 1.755 | 1.624 | 1.509 |
| 13 | 2.589 | 2.334 | 2.118 | 1.933 | 1.774 | 1.638 | 1.519 |
| 14 | 2.644 | 2.373 | 2.145 | 1.952 | 1.788 | 1.647 | 1.526 |
| 15 | 2.689 | 2.403 | 2.165 | 1.966 | 1.797 | 1.654 | 1.530 |
| 16 | 2.725 | 2.426 | 2.180 | 1.976 | 1.804 | 1.658 | 1.533 |
| 17 | 2.753 | 2.444 | 2.191 | 1.983 | 1.808 | 1.661 | 1.535 |
| 18 | 2.776 | 2.458 | 2.200 | 1.988 | 1.811 | 1.663 | 1.536 |
| 19 | 2.793 | 2.468 | 2.206 | 1.991 | 1.814 | 1.664 | 1.537 |
| 20 | 2.808 | 2.476 | 2.210 | 1.994 | 1.815 | 1.665 | 1.538 |
| 21 | 2.819 | 2.482 | 2.214 | 1.996 | 1.816 | 1.666 | 1.538 |
| 22 | 2.827 | 2.487 | 2.216 | 1.997 | 1.817 | 1.666 | 1.538 |
| 23 | 2.834 | 2.490 | 2.218 | 1.998 | 1.817 | 1.666 | |
| 24 | 2.839 | 2.493 | 2.219 | 1.999 | 1.818 | 1.666 | |
| 25 | 2.843 | 2.494 | 2.220 | 1.999 | | 1.660 | |
| 30 | 2.853 | 2.499 | 2.222 | 2.000 | | 1.667 | |
| 35 | 2.856 | 2.500 | | | | | |
| 40 | 2.857 | | | | | | |
| 45 | 2.857 | | | | | | |
| 50 | 2.857 | | | | | | |

## Gradient Factor - Uniform Equivalent - Annual Compounding

| n | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.3704 | 0.3636 | 0.3571 | 0.3509 | 0.3448 | 0.3390 | 0.3333 |
| 3 | 0.6619 | 0.6451 | 0.6291 | 0.6138 | 0.5991 | 0.5850 | 0.5714 |
| 4 | 0.8845 | 0.8559 | 0.8288 | 0.8031 | 0.7787 | 0.7554 | 0.7333 |
| 5 | 1.050 | 1.009 | 0.9706 | 0.9346 | 0.9007 | 0.8688 | 0.8387 |
| 6 | 1.169 | 1.117 | 0.1068 | 1.023 | 0.9808 | 0.9415 | 0.9048 |
| 7 | 1.254 | 1.191 | 1.134 | 1.081 | 1.032 | 0.9867 | 0.9449 |
| 8 | 1.312 | 1.241 | 1.177 | 1.118 | 1.064 | 1.014 | 0.9686 |
| 9 | 1.352 | 1.274 | 1.204 | 1.141 | 1.083 | 1.031 | 0.9824 |
| 10 | 1.379 | 1.296 | 1.222 | 1.155 | 1.095 | 1.040 | 0.9902 |
| 11 | 1.396 | 1.310 | 1.233 | 1.164 | 1.102 | 1.046 | 0.9946 |
| 12 | 1.408 | 1.319 | 1.240 | 1.169 | 1.106 | 1.049 | 0.9971 |
| 13 | 1.415 | 1.324 | 1.244 | 1.172 | 1.108 | 1.050 | 0.9984 |
| 14 | 1.420 | 1.328 | 1.246 | 1.174 | 1.109 | 1.051 | 0.9991 |
| 15 | 1.423 | 1.330 | 1.248 | 1.175 | 1.110 | 1.052 | 0.9995 |
| 16 | 1.425 | 1.331 | 1.249 | 1.176 | 1.111 | 1.052 | 0.9998 |
| 17 | 1.427 | 1.332 | 1.249 | | | 1.052 | 0.9999 |
| 18 | 1.427 | 1.333 | 1.250 | | | 1.052 | 0.9999 |
| 19 | 1.428 | | | | | | 1.000 |
| 20 | 1.428 | | | | | | |
| 21 | 1.428 | | | | | | |
| 22 | 1.428 | | | | | | |
| 23 | 1.428 | | | | | | |
| 24 | 1.429 | | | | | | |
| 25 | | | | | | | |
| 30 | | | | | | | |
| 35 | | | | | | | |
| 40 | | | | | | | |
| 45 | | | | | | | |
| 50 | | | | | | | |

## Gradient Factor - Present Worth - Annual Compounding

| | 2% | 4% | 6% | 8% | 10% | 12% | 14% | 16% | 18% |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9612 | 0.9246 | 0.8900 | 0.8573 | 0.8264 | 0.7972 | 0.7695 | 0.7432 | 0.7182 |
| 3 | 2.846 | 2.703 | 2.569 | 2.445 | 2.329 | 2.221 | 2.119 | 2.024 | 1.935 |
| 4 | 5.617 | 5.267 | 4.946 | 4.650 | 4.378 | 4.127 | 3.896 | 3.681 | 3.483 |
| 5 | 9.240 | 8.555 | 7.935 | 7.372 | 6.862 | 6.397 | 5.973 | 5.586 | 5.231 |
| 6 | 13.68 | 12.51 | 11.46 | 10.52 | 9.684 | 8.930 | 8.251 | 7.638 | 7.083 |
| 7 | 18.90 | 17.07 | 15.45 | 14.02 | 12.76 | 11.64 | 10.65 | 9.761 | 8.967 |
| 8 | 24.88 | 22.18 | 19.84 | 17.81 | 16.06 | 14.47 | 13.10 | 11.90 | 10.83 |
| 9 | 31.57 | 27.80 | 24.58 | 21.81 | 19.42 | 17.36 | 15.56 | 14.00 | 12.63 |
| 10 | 38.96 | 33.88 | 29.60 | 25.98 | 22.89 | 20.25 | 17.99 | 16.04 | 14.35 |
| 11 | 47.00 | 40.38 | 34.87 | 30.27 | 26.40 | 23.13 | 20.36 | 17.99 | 15.97 |
| 12 | 55.67 | 47.25 | 40.34 | 34.63 | 29.90 | 25.95 | 22.64 | 19.85 | 17.48 |
| 13 | 64.94 | 54.45 | 45.96 | 39.05 | 33.38 | 28.70 | 24.82 | 21.59 | 18.88 |
| 14 | 74.80 | 61.96 | 51.71 | 43.47 | 36.80 | 31.36 | 26.90 | 23.22 | 20.16 |
| 15 | 85.20 | 69.74 | 57.55 | 47.89 | 40.15 | 33.92 | 28.86 | 24.73 | 21.33 |
| 16 | 90.13 | 77.74 | 63.46 | 52.26 | 43.42 | 36.37 | 30.71 | 26.12 | 22.39 |
| 17 | 107.6 | 85.96 | 69.40 | 56.59 | 46.58 | 38.70 | 32.43 | 27.41 | 23.35 |
| 18 | 119.5 | 94.35 | 75.36 | 60.84 | 49.64 | 40.91 | 34.04 | 28.58 | 24.21 |
| 19 | 131.8 | 102.9 | 81.31 | 65.01 | 52.58 | 43.00 | 35.53 | 29.66 | 24.99 |
| 20 | 144.6 | 111.6 | 87.23 | 69.09 | 55.41 | 44.97 | 36.91 | 30.63 | 25.68 |
| 21 | 151.8 | 120.3 | 93.11 | 73.06 | 58.11 | 46.82 | 38.19 | 31.52 | 26.30 |
| 22 | 171.4 | 129.2 | 98.94 | 76.93 | 60.69 | 48.55 | 39.37 | 32.32 | 26.85 |
| 23 | 185.3 | 138.1 | 104.7 | 80.67 | 63.15 | 50.18 | 40.45 | 33.04 | 27.34 |
| 24 | 199.6 | 147.1 | 110.4 | 84.30 | 65.48 | 51.69 | 41.44 | 33.70 | 27.77 |
| 25 | 214.2 | 156.1 | 116.0 | 87.80 | 67.70 | 53.10 | 42.34 | 34.28 | 28.16 |
| 30 | 291.7 | 201.1 | 142.4 | 103.4 | 77.08 | 58.78 | 45.81 | 36.42 | 29.49 |
| 35 | 374.9 | 244.9 | 165.7 | 116.1 | 83.99 | 62.61 | 47.96 | 37.63 | 30.18 |
| 40 | 462.0 | 286.5 | 186.0 | 126.0 | 88.95 | 65.12 | 49.24 | 38.30 | 30.53 |
| 45 | 551.6 | 325.4 | 203.1 | 133.7 | 92.45 | 66.73 | 50.00 | 38.66 | 30.70 |
| 50 | 642.4 | 361.2 | 217.5 | 139.6 | 94.89 | 67.76 | 50.44 | 38.85 | 30.79 |

## Gradient Factor - Present Worth - Annual Compounding

| | 20% | 25% | 30% | 35% | 40% | 45% | 50% | 55% | 60% |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.6944 | 0.6340 | 0.5917 | 0.5487 | 0.5102 | 0.4756 | 0.4444 | 0.4162 | 0.3906 |
| 3 | 1.852 | 1.664 | 1.502 | 1.362 | 1.239 | 1.132 | 1.037 | 0.9533 | 0.8789 |
| 4 | 3.299 | 2.893 | 2.552 | 2.265 | 2.020 | 1.810 | 1.630 | 1.473 | 1.337 |
| 5 | 4.906 | 4.204 | 3.623 | 3.157 | 2.764 | 2.434 | 2.156 | 1.920 | 1.718 |
| 6 | 6.581 | 5.514 | 4.666 | 3.983 | 3.428 | 2.972 | 2.595 | 2.281 | 2.016 |
| 7 | 8.255 | 6.773 | 5.622 | 4.717 | 3.997 | 3.418 | 2.947 | 2.560 | 2.240 |
| 8 | 9.883 | 7.947 | 6.480 | 5.352 | 4.471 | 3.776 | 3.220 | 2.770 | 2.403 |
| 9 | 11.43 | 9.021 | 7.234 | 5.889 | 4.858 | 4.058 | 3.428 | 2.925 | 2.519 |
| 10 | 12.89 | 9.987 | 7.887 | 6.336 | 5.170 | 4.277 | 3.584 | 3.037 | 2.601 |
| 11 | 14.23 | 10.85 | 8.445 | 6.705 | 5.417 | 4.445 | 3.699 | 3.118 | 2.658 |
| 12 | 15.47 | 11.60 | 8.917 | 7.005 | 5.611 | 4.527 | 3.784 | 3.175 | 2.697 |
| 13 | 16.59 | 12.26 | 9.314 | 7.247 | 5.762 | 4.668 | 3.846 | 3.215 | 2.724 |
| 14 | 17.60 | 12.83 | 9.644 | 7.442 | 5.879 | 4.740 | 3.890 | 3.244 | 2.742 |
| 15 | 18.51 | 13.33 | 9.917 | 7.597 | 5.969 | 4.293 | 3.922 | 3.263 | 2.754 |
| 16 | 19.32 | 13.75 | 10.14 | 7.721 | 6.038 | 4.832 | 3.945 | 3.277 | 2.762 |
| 17 | 20.04 | 14.11 | 10.33 | 7.818 | 6.090 | 4.861 | 3.961 | 3.286 | 2.767 |
| 18 | 20.68 | 14.41 | 10.48 | 7.895 | 6.130 | 4.882 | 3.973 | 3.292 | 2.771 |
| 19 | 21.24 | 14.67 | 10.60 | 7.955 | 6.160 | 4.898 | 3.981 | 3.297 | 2.773 |
| 20 | 21.74 | 14.89 | 10.70 | 8.002 | 6.183 | 4.909 | 3.987 | 3,300 | 2.775 |
| 21 | 22.17 | 15.08 | 10.78 | 8.038 | 6.200 | 4.917 | 3.991 | 3.302 | 2.776 |
| 22 | 22.55 | 15.23 | 10.85 | 8.067 | 6.213 | 4.923 | 3.994 | 3.303 | 2.777 |
| 23 | 22.89 | 15.36 | 10.90 | 8.089 | 6.222 | 4.927 | 3.996 | 3.304 | 2.777 |
| 24 | 23.18 | 15.47 | 10.94 | 8.106 | 6.229 | 4.930 | 3.997 | 3.305 | 2.777 |
| 25 | 23.43 | 15.56 | 10.98 | 8.119 | 6.235 | 4.933 | 3.998 | 3.305 | 2.777 |
| 30 | 24.26 | 15.83 | 11.07 | 8.152 | 6.247 | 4.937 | 4.000 | 3.306 | 2.778 |
| 35 | 24.66 | 15.94 | 11.10 | 8.160 | 6.249 | 4.938 | ↓ | ↓ | ↓ |
| 40 | 24.85 | 15.98 | 11.11 | 8.163 | 6.250 | ↓ | | | |
| 45 | 24.93 | 15.99 | ↓ | ↓ | ↓ | | | | |
| 50 | 24.97 | 16.00 | | | | | | | |

## Gradient Factor - Present Worth - Annual Compounding

| | 65% | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.3673 | 0.3460 | 0.3265 | 0.3086 | 0.2922 | 0.2770 | 0.2630 | 0.2500 |
| 3 | 0.8125 | 0.7531 | .6997 | 0.6516 | 0.6081 | 0.5686 | 0.5327 | 0.5000 |
| 4 | 1.217 | 1.112 | 1.020 | 0.9374 | 0.8642 | 0.7988 | 0.7402 | 0.6875 |
| 5 | 1.544 | 1.394 | 1.263 | 1.149 | 1.049 | 0.9603 | 0.8821 | 0.8125 |
| 6 | 1.792 | 1.601 | 1.437 | 1.296 | 1.173 | 1.067 | 0.9730 | 0.8906 |
| 7 | 1.972 | 1.747 | 1.557 | 1.394 | 1.254 | 1.134 | 1.029 | 0.9375 |
| 8 | 2.100 | 1.848 | 1.636 | 1.476 | 1.305 | 1.175 | 1.062 | 0.9648 |
| 9 | 2.188 | 1.915 | 1.688 | 1.498 | 1.337 | 1.200 | 1.082 | 0.9805 |
| 10 | 2.248 | 1.960 | 1.722 | 1.523 | 1.356 | 1.214 | 1.093 | 0.9893 |
| 11 | 2.289 | 1.989 | 1.743 | 1.539 | 1.368 | 1.223 | 1.100 | 0.9941 |
| 12 | 2.316 | 2.008 | 1.756 | 1.548 | 1.374 | 1.228 | 1.103 | 0.9968 |
| 13 | 2.334 | 2.020 | 1.765 | 1.554 | 1.378 | 1.231 | 1.106 | 0.9983 |
| 14 | 2.345 | 2.028 | 1.770 | 1.557 | 1.381 | 1.232 | 1.107 | 0.9991 |
| 15 | 2.353 | 2.033 | 1.773 | 1.559 | 1.382 | 1.233 | 1.107 | 0.9995 |
| 16 | 2.358 | 2.036 | 1.775 | 1.561 | 1.383 | 1.234 | 1.108 | 0.9997 |
| 17 | 2.361 | 2.038 | 1.776 | 1.561 | 1.384 | 1.234 | | 0.9999 |
| 18 | 2.363 | 2.039 | 1.777 | 1.562 | | 1.234 | | 0.9999 |
| 19 | 2.365 | 2.040 | 1.777 | 1.562 | | 1.235 | | 1.000 |
| 20 | 2.365 | 2.040 | 1.777 | 1.562 | | | | |
| 21 | 2.366 | 2.040 | 1.778 | 1.562 | | | | |
| 22 | 2.366 | 2.041 | | 1.562 | | | | |
| 23 | 2.366 | | | 1.562 | | | | |
| 24 | 2.367 | | | 1.562 | | | | |
| 25 | | | | 1.562 | | | | |
| 30 | | | | 1.563 | | | | |
| 35 | | | | | | | | |
| 40 | | | | | | | | |
| 45 | | | | | | | | |
| 50 | | | | | | | | |

## Present Worth Factor - Single Payment - Continuous Compounding

| Years | Months | 2% | 4% | 6% | 8% | 10% |
|---|---|---|---|---|---|---|
| 0 | 1 | 0.9983 | 0.9967 | 0.9950 | 0.9934 | 0.9911 |
| | 2 | 0.9967 | 0.9934 | 0.9900 | 0.9868 | 0.9835 |
| | 3 | 0.9950 | 0.9900 | 0.9851 | 0.9802 | 0.9753 |
| | 4 | 0.9933 | 0.9868 | 0.9802 | 0.9737 | 0.9672 |
| | 5 | 0.9917 | 0.9835 | 0.9753 | 0.9672 | 0.9592 |
| | 6 | 0.9900 | 0.9802 | 0.9704 | 0.9608 | 0.9512 |
| | 7 | 0.9884 | 0.0769 | 0.9656 | 0.9544 | 0.9433 |
| | 3 | 0.9868 | 0.9737 | 0.9608 | 0.9481 | 0.9355 |
| | 9 | 0.9851 | 0.9704 | 0.9560 | 0.9418 | 0.9277 |
| | 10 | 0.9835 | 0.9762 | 0.9512 | 0.9355 | 0.9200 |
| | 11 | 0.9818 | 019640 | 0.9465 | 0.9293 | 0.9124 |
| 1 | 0 | 0.9802 | 0.9608 | 0.9418 | 0.9231 | 0.9048 |
| | 3 | 0.9753 | 0.9512 | 0.9277 | 0.9048 | 0.8624 |
| | 6 | 0.9704 | 0.9418 | 0.9139 | 0.8869 | 0.8607 |
| | 9 | 0.9656 | 0.9324 | 0.9003 | 0.8694 | 0.8395 |
| 2 | 0 | 0.9608 | 0.9321 | 0.8869 | 0.8521 | 0.8187 |
| | 3 | 0.9560 | 0.9139 | 0.8737 | 0.8353 | 0.7985 |
| | 6 | 0.9512 | 0.9048 | 0.8607 | 0.8187 | 0.7788 |
| | 9 | 0.9465 | 0.8958 | 0.8479 | 0.8075 | 0.7596 |
| 3 | 0 | 0.9418 | 0.8869 | 0.8352 | 0.7866 | 0.7408 |
| | 3 | 0.9371 | 0.8781 | 0.8228 | 0.7711 | 0.7225 |
| | 6 | 0.9324 | 0.8694 | 0.8106 | 0.7558 | 0.7047 |
| | 9 | 0.9277 | 0.8607 | 0.7985 | 0.7408 | 0.6873 |
| 4 | 0 | 0.9231 | 0.8521 | 0.7866 | 0.7261 | 0.6703 |
| | 3 | 0.9185 | 0.8437 | 0.7749 | 0.7118 | 0.6538 |
| | 6 | 0.9139 | 0.8353 | 0.7634 | 0.6977 | 0.6376 |
| | 9 | 0.9094 | 0.8270 | 0.7520 | 0.6839 | 0.6219 |
| 5 | 0 | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 |
| | 3 | 0.9003 | 0.8106 | 0.7298 | 0.6570 | 0.5916 |
| | 6 | 0.8958 | 0.8025 | 0.7189 | 0.6440 | 0.5769 |
| | 9 | 0.8914 | 0.7945 | 0.7082 | 0.6313 | 0.5627 |
| 6 | | 0.8869 | 0.7866 | 0.6977 | 0.6188 | 0.5488 |
| 7 | | 0.8694 | 0.7558 | 0.6570 | 0.5712 | 0.4966 |
| 8 | | 0.8521 | 0.7261 | 0.6188 | 0.5273 | 0.4493 |
| 9 | | 0.8353 | 0.6977 | 0.5827 | 0.4868 | 0.4066 |
| 10 | | 0.8187 | 0.6703 | 0.5488 | 0.4493 | 0.3679 |

## Present Worth Factor - Single Payment - Continuous Compounding

| Years | Months | 12% | 14% | 16% | 18% | 20% |
|---|---|---|---|---|---|---|
| 0 | 1 | 0.9900 | 0.9884 | 0.9867 | 0.9851 | 0.9835 |
| | 2 | 0.9801 | 0.9769 | 0.9737 | 0.9704 | 0.9672 |
| | 3 | 0.9704 | 0.9656 | 0.9608 | 0.9560 | 0.9512 |
| | 4 | 0.9608 | 0.9544 | 0.9481 | 0.9418 | 0.9355 |
| | 5 | 0.9512 | 0.9433 | 0.9355 | 0.9277 | 0.9200 |
| | 6 | 0.9418 | 0.9324 | 0.9231 | 0.9139 | 0.9048 |
| | 7 | 0.9324 | 0.9216 | 0.9109 | 0.9003 | 0.8899 |
| | 8 | 0.9231 | 0.9109 | 0.8988 | 0.8869 | 0.8752 |
| | 9 | 0.9139 | 0.9003 | 0.8869 | 0.8737 | 0.8607 |
| | 10 | 0.9048 | 0.8899 | 0.8752 | 0.8607 | 0.8465 |
| | 11 | 0.8958 | 0.8796 | 0.8636 | 0.8479 | 0.8325 |
| 1 | 0 | 0.8869 | 0.8694 | 0.8521 | 0.8353 | 0.8187 |
| | 3 | 0.8607 | 0.8395 | 0.8187 | 0.7985 | 0.7788 |
| | 6 | 0.8353 | 0.8105 | 0.7866 | 0.7634 | 0.7408 |
| | 9 | 0.8106 | 0.7827 | 0.7558 | 0.7298 | 0.7047 |
| 2 | 0 | 0.7866 | 0.7558 | 0.7261 | 0.6977 | 0.6703 |
| | 3 | 0.7634 | 0.7298 | 0.6977 | 0.6670 | 0.6376 |
| | 6 | 0.7408 | 0.7047 | 0.6703 | 0.6376 | -.6065 |
| | 9 | 0.7189 | 0.6805 | 0.6440 | 0.6096 | 0.5769 |
| 3 | 0 | 0.6977 | 0.6570 | 0.6187 | 0.5827 | 0.5488 |
| | 3 | 0.6770 | 0.6344 | 0.5945 | 0.5571 | 0.5220 |
| | 6 | 0.6570 | 0.6126 | 0.5712 | 0.5326 | 0.4966 |
| | 9 | 0.6376 | 0.5916 | 0.5488 | 0.5091 | 0.4724 |
| 4 | 0 | 0.6187 | 0.5712 | 0.5273 | 0.4868 | 0.4493 |
| | 3 | 0.6005 | 0.5516 | 0.5066 | 0.4653 | 0.4274 |
| | 6 | 0.5827 | 0.5326 | 0.4867 | 0.4449 | 0.4066 |
| | 9 | 0.5655 | 0.5143 | 0.4677 | 0.4253 | 0.3867 |
| 5 | 0 | 0.5488 | 0.4966 | 0.4493 | 0.4066 | 0.3679 |
| | 3 | 0.5326 | 0.4795 | 0.4317 | 0.3887 | 0.3499 |
| | 6 | 0.5169 | 0.4630 | 0.4148 | 0.3716 | 0.3329 |
| | 9 | 0.5016 | 0.4471 | 0.3985 | 0.3552 | 0.3166 |
| 6 | | 0.4868 | 0.4317 | 0.3829 | 0.3396 | 0.3012 |
| 7 | | 0.4317 | 0.3753 | 0.3263 | 0.2837 | 0.2466 |
| 8 | | 0.3829 | 0.3263 | 0.2870 | 0.2369 | 0.2019 |
| 9 | | 0.3396 | 0.2837 | 0.2369 | 0.1979 | 0.1653 |
| 10 | | 0.3012 | 0.2466 | 0.2019 | 0.1653 | 0.1353 |

## Present Worth Factor - Single Payment - Continuous Compounding

| Years | Months | 25% | 30% | 35% | 40% | 45% |
|---|---|---|---|---|---|---|
| 0 | 1 | 0.9738 | 0.9753 | 0.9713 | 0.9672 | 0.9632 |
| | 2 | 0.9592 | 0.9512 | 0.9433 | 0.9355 | 0.9277 |
| | 3 | 0.9394 | 0.9277 | 0.9162 | 0.9048 | 0.8936 |
| | 4 | 0.9200 | 0.9048 | 0.8899 | 0.8752 | 0.8607 |
| | 5 | 0.9011 | 0.8825 | 0.8648 | 0.8465 | 0.8290 |
| | 6 | 0.8825 | 0.8607 | 0.8395 | 0.8187 | 0.7985 |
| | 7 | 0.8643 | 0.8395 | 0.8153 | 0.7919 | 0.7691 |
| | 8 | 0.8465 | 0.8187 | 0.7919 | 0.7659 | 0.7408 |
| | 9 | 0.8290 | 0.7985 | 0.7691 | 0.7408 | 0.7136 |
| | 10 | 0.8119 | 0.7788 | 0.7470 | 0.7165 | 0.6873 |
| | 11 | 0.7951 | 0.7596 | 0.7255 | 0.6930 | 0.6620 |
| 1 | 0 | 0.7788 | 0.7408 | 0.7047 | 0.6703 | 0.6376 |
| | 3 | 0.7316 | 0.6873 | 0.6456 | 0.6065 | 0.5698 |
| | 6 | 0.6873 | 0.6376 | 0.5916 | 0.5488 | 0.5092 |
| | 9 | 0.6456 | 0.5916 | 0.5420 | 0.4966 | 0.4550 |
| 2 | 0 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 |
| | 3 | 0.5698 | 0.5092 | 0.4550 | 0.4066 | 0.3633 |
| | 6 | 0.5353 | 0.4724 | 0.4169 | 0.3679 | 0.3247 |
| | 9 | 0.5028 | 0.4382 | 0.3819 | 0.3329 | 0.2901 |
| 3 | 0 | 0.4724 | 0.4065 | 0.3499 | 0.3012 | 0.2592 |
| | 3 | 0.4437 | 0.3772 | 0.3206 | 0.2725 | 0.2317 |
| | 6 | 0.4169 | 0.3499 | 0.2938 | 0.2466 | 0.2070 |
| | 9 | 0.3916 | 0.3247 | 0.2691 | 0.2231 | 0.1850 |
| 4 | 0 | 0.3679 | 0.3012 | 0.2466 | 0.2019 | 0.1653 |
| | 3 | 0.3456 | 0.2794 | 0.2259 | 0.1827 | 0.1477 |
| | 6 | 0.3247 | 0.2592 | 0.2070 | 0.1653 | 0.1320 |
| | 9 | 0.3050 | 0.2405 | 01897 | 0.1496 | 0.1179 |
| 5 | 0 | 0.2865 | 0.2231 | 0.1738 | 0.1353 | 0.1054 |
| | 3 | 0.2691 | 0.2070 | 0.1592 | 0.1225 | 0.0942 |
| | 6 | 0.2528 | 0.1920 | 0.1459 | 0.1108 | 0.0842 |
| | 9 | 0.2375 | 0.1781 | 0.1337 | 0.1003 | 0.0752 |
| 6 | | 0.2231 | 0.1653 | 0.1225 | 0.0907 | 0.0672 |
| 7 | | 0.1738 | 0.1225 | 0.0863 | 0.0608 | 0.0429 |
| 8 | | 0.1353 | 0.0907 | 0.0608 | 0.0408 | 0.0273 |
| 9 | | 0.1054 | 0.0672 | 0.0429 | 0.0273 | 0.0174 |
| 10 | | 0.0821 | 0.0498 | 0.0302 | 0.0183 | 0.0111 |

## Present Worth Factor - Single Payment - Continuous Compounding

| Years | Months | 50% | 55% | 60% | 65% |
|---|---|---|---|---|---|
| 0 | 1 | 0.9592 | 0.9552 | 0.9512 | 0.9473 |
| | 2 | 0.9200 | 0.9124 | 0.9048 | 0.8973 |
| | 3 | 0.8825 | 0.8715 | 0.8606 | 0.8500 |
| | 4 | 0.8465 | 0.8325 | 0.8187 | 0.8052 |
| | 5 | 0.8119 | 0.7952 | 0.7788 | 0.7627 |
| | 6 | 0.7788 | 0.7596 | 0.7408 | 0.7225 |
| | 7 | 0.7470 | 0.7255 | 0.7047 | 0.6844 |
| | 8 | 0.7165 | 0.6930 | 0.6703 | 0.6483 |
| | 9 | 0.6873 | 0.6620 | 0.6376 | 0.6141 |
| | 10 | 0.6592 | 0.6323 | 0.6065 | 0.5817 |
| | 11 | 0.6323 | 0.6040 | 0.5769 | 0.5511 |
| 1 | 0 | 0.6065 | 0.5769 | 0.5488 | 0.5220 |
| | 3 | 0.5353 | 0.5028 | 0.4724 | 0.4437 |
| | 6 | 0.4724 | 0.4382 | 0.4066 | 0.3772 |
| | 9 | 0.4169 | 0.3819 | 0.3499 | 0.3206 |
| 2 | 0 | 0.3679 | 0.3329 | 0.3012 | 0.2725 |
| | 3 | 0.3247 | 0.2901 | 0.2592 | 0.2317 |
| | 6 | 0.2865 | 0.2528 | 0.2231 | 0.1969 |
| | 9 | 0.2528 | 0.2204 | 0.1920 | 0.1674 |
| 3 | 0 | 0.2231 | 0.1920 | 0.1653 | 0.1423 |
| | 3 | 0.1969 | 0.1674 | 0.1423 | 0.1209 |
| | 6 | 0.1738 | 0.1459 | 0.1225 | 0.1027 |
| | 9 | 0.1534 | 0.1271 | 0.1054 | 0.0874 |
| 4 | 0 | 0.1353 | 0.1108 | 0.0907 | 0.0743 |
| | 3 | 0.1194 | 0.0966 | 0.0780 | 0.0631 |
| | 6 | 0.1054 | 0.0841 | 0.0672 | 0.0537 |
| | 9 | 0.0930 | 0.0734 | 0.0578 | 0.0456 |
| 5 | 0 | 0.0821 | 0.0639 | 0.0498 | 0.0388 |
| | 3 | 0.0724 | 0.0557 | 0.0429 | 0.0330 |
| | 6 | 0.0639 | 0.0486 | 0.0369 | 0.0280 |
| | 9 | 0.0564 | 0.0423 | 0.0317 | 0.0238 |
| 6 | | 0.0498 | 0.0369 | 0.0273 | 0.0202 |
| 7 | | 0.0302 | 0.0212 | 0.0150 | 0.0106 |
| 8 | | 0.0183 | 0.0123 | 0.0082 | 0.0055 |
| 9 | | 0.0111 | 0.0071 | 0.0045 | 0.0029 |
| 10 | | 0.0067 | 0.0041 | 0.0025 | 0.0015 |

## Present Worth Factor - Single Payment - Continuous Compounding

| Yr. | Mo. | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
|---|---|---|---|---|---|---|---|---|
| | 1 | 0.9433 | 0.9394 | 0.9355 | 0.9316 | 0.9277 | 0.9239 | 0.9200 |
| | 2 | 0.8899 | 0.8824 | 0.8752 | 0.8679 | 0.8607 | 0.8535 | 0.8464 |
| | 3 | 0.8395 | 0.8290 | 0.8187 | 0.8086 | 0.7985 | 0.7886 | 0.7788 |
| | 4 | 0.7919 | 0.7788 | 0.7659 | 0.7533 | 0.7408 | 0.7286 | 0.7165 |
| | 5 | 0.7470 | 0.7316 | 0.7165 | 0.7018 | 0.6873 | 0.6731 | 0.6592 |
| | 6 | 0.7046 | 0.6873 | 0.6703 | 0.6538 | 0.6376 | 0.6219 | 0.6065 |
| | 7 | 0.6648 | 0.6456 | 0.6271 | 0.6091 | 0.5916 | 0.5746 | 0.5580 |
| | 8 | 0.6271 | 0.6065 | 0.5866 | 0.5674 | 0.5488 | 0.5308 | 0.5134 |
| | 9 | 0.5916 | 0.5698 | 0.5488 | 0.5286 | 0.5091 | 0.4904 | 0.4724 |
| | 10 | 0.5580 | 0.5352 | 0.5134 | 0.4925 | 0.4724 | 0.4531 | 0.4346 |
| | 11 | 0.5264 | 0.5028 | 0.4803 | 0.4588 | 0.4382 | 0.4186 | 0.3998 |
| 1 | 0 | 0.4966 | 0.4724 | 0.4493 | 0.4274 | 0.4066 | 0.3867 | 0.3679 |
| | 3 | 0.4169 | 0.3916 | 0.3679 | 0.3456 | 0.3247 | 0.3050 | 0.2865 |
| | 6 | 0.3499 | 0.3247 | 0.3012 | 0.2794 | 0.2592 | 0.2405 | 0.2231 |
| | 9 | 0.2938 | 0.2691 | 0.2466 | 0.2259 | 0.2070 | 0.1897 | 0.1738 |
| 2 | 0 | 0.2466 | 0.2231 | 0.2019 | 0.1826 | 0.1653 | 0.1496 | 0.1353 |
| | 3 | 0.2070 | 0.1850 | 0.1653 | 0.1477 | 0.1320 | 0.1179 | 0.1054 |
| | 6 | 0.1738 | 0.1533 | 0.1353 | 0.1194 | 0.1054 | 0.0930 | 0.0821 |
| | 9 | 0.1459 | 0.1271 | 0.1108 | 0.0966 | 0.0841 | 0.0734 | 0.0639 |
| 3 | 0 | 0.1225 | 0.1054 | 0.0907 | 0.0781 | 0.0672 | 0.0578 | 0.0498 |
| | 3 | 0.1028 | 0.0874 | 0.0743 | 0.0631 | 0.0537 | 0.0456 | 0.0388 |
| | 6 | 0.0863 | 0.0724 | 0.0608 | 0.0510 | 0.0429 | 0.0360 | 0.0302 |
| | 9 | 0.0724 | 0.0601 | 0.0498 | 0.0413 | 0.0342 | 0.0284 | 0.0235 |
| 4 | 0 | 0.0608 | 0.0498 | 0.0403 | 0.0334 | 0.0273 | 0.0224 | 0.0183 |
| | 3 | 0.0510 | 0.0413 | 0.0334 | 0.0270 | 0.0218 | 0.0176 | 0.0143 |
| | 6 | 0.0429 | 0.0342 | 0.0273 | 0.0218 | 0.0174 | 0.0139 | 0.0111 |
| | 9 | 0.0360 | 0.0284 | 0.0224 | 0.0176 | 0.0139 | 0.0110 | 0.0087 |
| 5 | 0 | 0.0302 | 0.0235 | 0.0183 | 0.0143 | 0.0111 | 0.0086 | 0.0067 |
| | 3 | 0.0253 | 0.0195 | 0.0150 | 0.0115 | 0.0089 | 0.0068 | 0.0052 |
| | 6 | 0.0213 | 0.0162 | 0.0123 | 0.0093 | 0.0071 | 0.0054 | 0.0041 |
| | 9 | 0.0179 | 0.0134 | 0.0101 | 0.0075 | 0.0057 | 0.0042 | 0.0032 |
| 6 | | 0.0150 | 0.0111 | 0.0082 | 0.0060 | 0.0045 | 0.0033 | 0.0025 |
| 7 | | 0.0074 | 0.0052 | 0.0037 | 0.0026 | 0.0018 | 0.0013 | 0.0009 |
| 8 | | 0.0037 | 0.0025 | 0.0017 | 0.0011 | 0.0007 | 0.0005 | 0.0003 |
| 9 | | 0.0018 | 0.0012 | 0.0007 | 0.0005 | 0.0003 | 0.0002 | 0.0001 |
| 10 | | 0.0009 | 0.0006 | 0.0003 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |

## Present Worth Factor - Single Payment - Continuous Compounding

| Yrs. | Mo. | 2% | 4% | 6% | 8% | 10% | 12% | 14% | 16% | 18% | 20% |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | | 0.8025 | 0.6440 | 0.5169 | 0.4148 | 0.3329 | 0.2671 | 0.2144 | 0.1720 | 0.1380 | 0.1108 |
| 12 | | 0.7866 | 0.6188 | 0.4868 | 0.3829 | 0.3012 | 0.2369 | 0.1864 | 0.1466 | 0.1153 | 0.0907 |
| 13 | | 0.7710 | 0.5945 | 0.4584 | 0.3535 | 0.2725 | 0.2101 | 0.1620 | 0.1249 | 0.0963 | 0.0743 |
| 14 | | 0.7557 | 0.5712 | 0.4317 | 0.3263 | 0.2466 | 0.1864 | 0.1409 | 0.1065 | 0.0805 | 0.0608 |
| 15 | | 0.7408 | 0.5488 | 0.4066 | 0.3012 | 0.2231 | 0.1653 | 0.1225 | 0.0907 | 0.0672 | 0.0493 |
| 16 | | 0.7261 | 0.5273 | 0.3829 | 0.2780 | 0.2019 | 0.1466 | 0.1065 | 0.0773 | 0.0561 | 0.0408 |
| 17 | | 0.7118 | 0.5066 | 0.3606 | 0.2567 | 0.1827 | 0.1300 | 0.0926 | 0.0659 | 0.0469 | 0.0334 |
| 18 | | 0.6977 | 0.4868 | 0.3395 | 0.2369 | 0.1653 | 0.1153 | 0.0805 | 0.0561 | 0.0392 | 0.0272 |
| 19 | | 0.6838 | 0.4677 | 0.3198 | 0.2187 | 0.1496 | 0.1023 | 0.0699 | 0.0478 | 0.0327 | 0.0224 |
| 20 | | 0.6703 | 0.4493 | 0.3012 | 0.2019 | 0.1353 | 0.0907 | 0.0608 | 0.0408 | 0.0273 | 0.0183 |
| 21 | | 0.6570 | 0.4317 | 0.2873 | 0.1864 | 0.1225 | 0.0805 | 0.0529 | 0.0347 | 0.0228 | 0.0150 |
| 22 | | 0.6440 | 0.4148 | 0.2671 | 0.1720 | 0.1108 | 0.0714 | 0.0460 | 0.0296 | 0.0190 | 0.0123 |
| 23 | | 0.6313 | 0.3985 | 0.2516 | 0.1588 | 0.1003 | 0.0633 | 0.0400 | 0.0152 | 0.0159 | 0.0101 |
| 24 | | 0.6188 | 0.3829 | 0.2369 | 0.1466 | 0.0907 | 0.0561 | 0.0347 | 0.0215 | 0.0133 | 0.0082 |
| 25 | | 0.6065 | 0.3679 | 0.2231 | 0.1353 | 0.0821 | 0.0498 | 0.0302 | 0.0183 | 0.0111 | 0.0067 |
| 30 | | 0.5488 | 0.3012 | 0.1653 | 0.0907 | 0.0498 | 0.0273 | 0.0150 | 0.0082 | 0.0045 | 0.0025 |
| 35 | | 0.4966 | 0.2465 | 0.1224 | 0.0608 | 0.0302 | 0.0150 | 0.0074 | 0.0037 | 0.0018 | 0.0009 |
| 40 | | 0.4493 | 0.2019 | 0.0907 | 0.0408 | 0.0183 | 0.0082 | 0.0037 | 0.0017 | 0.0007 | 0.0003 |
| 45 | | 0.4066 | 0.1653 | 0.0672 | 0.0273 | 0.0111 | 0.0045 | 0.0018 | 0.0007 | 0.0003 | 0.0001 |
| 50 | | 0.3679 | 0.1353 | 0.0498 | 0.0183 | 0.0067 | 0.0025 | 0.0009 | 0.0003 | 0.0001 | 0.0000 |

## Present Worth Factor - Single Payment - Continuous Compounding

| Yr. | Mo. | 25% | 30% | 35% | 40% | 45% | 50% | 55% | 60% |
|---|---|---|---|---|---|---|---|---|---|
| 11 | | 0.0639 | 0.0369 | 0.0213 | 0.0123 | 0.0071 | 0.0041 | 0.0023 | 0.0014 |
| 12 | | 0.0498 | 0.0273 | 9.0150 | 0.0082 | 0.0045 | 0.0025 | 0.0014 | 0.0007 |
| 13 | | 0.0388 | 0.202 | 0.0106 | 0.0055 | 0.0029 | 0.0015 | 0.0008 | 0.0004 |
| 14 | | 0.0302 | 0.0150 | 0.0074 | 0.0037 | 0.0018 | 0.0009 | 0.0005 | 0.0002 |
| 15 | | 0.0235 | 0.0111 | 0.0052 | 0.0025 | 0.0012 | 0.0006 | 0.0003 | 0.0001 |
| 16 | | 0.0183 | 0.0082 | 0.0037 | 0.0017 | 0.0007 | 0.0003 | 0.0002 | 0.0001 |
| 17 | | 0.0143 | 0.0061 | 0.0026 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 |
| 18 | | 0.0111 | 0.0095 | 0.0018 | 0.0007 | 0.0003 | 0.0001 | 0.0001 | 0.0000 |
| 19 | | 0.0087 | 0.0033 | 0.0013 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | |
| 20 | | 0.0067 | 0.0025 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | | |
| 21 | | 0.0052 | 0.0018 | 0.0006 | 0.0002 | 0.0001 | | | |
| 22 | | 0.0041 | 010014 | 0.0005 | 0.0002 | 0.0001 | | | |
| 23 | | 0.0032 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | | | |
| 24 | | 0.0025 | 0.0007 | 0.0002 | 0.0001 | | | | |
| 25 | | 0.0019 | 0.0005 | 0.0002 | 0.0000 | | | | |
| 30 | | 0.0005 | 0.0001 | 0.0000 | | | | | |
| 35 | | 0.0002 | 0.0000 | | | | | | |
| 40 | | 0.0000 | | | | | | | |
| 45 | | | | | | | | | |
| 50 | | | | | | | | | |

## Present Worth Factor - Single Payment - Continuous Compounding

| Yr. | Mo | 65% | 70% | 75% | 80% | 85% | 90% | 95% | 100% |
|---|---|---|---|---|---|---|---|---|---|
| 11 | | 0.0007 | 0.0005 | 0.0003 | 0.0002 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 12 | | 0.0004 | 0.0002 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | | |
| 13 | | 0.0002 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | | |
| 14 | | 0.0001 | 0.0001 | 0.0000 | | | | | |
| 15 | | 0.0001 | 0.0000 | | | | | | |
| 16 | | 0.0000 | | | | | | | |
| 17 | | | | | | | | | |
| 18 | | | | | | | | | |
| 19 | | | | | | | | | |
| 20 | | | | | | | | | |
| 21 | | | | | | | | | |
| 22 | | | | | | | | | |
| 23 | | | | | | | | | |
| 24 | | | | | | | | | |
| 25 | | | | | | | | | |
| 30 | | | | | | | | | |
| 35 | | | | | | | | | |
| 40 | | | | | | | | | |
| 45 | | | | | | | | | |
| 50 | | | | | | | | | |